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THE APPLICATION OF FUZZY LOGIC IN ENGINEERING APPLICATIONS

In order to describe the phenomenon for which the mathematical model or input data are unknown, the fuzzy logic is applied. The fuzzy theory enables to find the most reliable solution on the assumption that the input data are fuzzed. This paper presents the possibility of application of fuzzy theory in engineering problems. The theoretical basis of the fuzzy logic and mathematical calculations on fuzzy variables are presented. The comparison of two methods used in fuzzy logic – extension principle and α -level optimization are written and compared. Examples of the application of aforementioned methods for solving simple engineering problem were presented. Numerical calculations were done with the use of MATLAB program. The selection of the most reliable solution, based on finding the mass center or with the use of rank level method, was also shown.

Keywords: fuzzy logic, α -level optimization, extension principle, mechanical engineering

1. Introduction

Precise information which is determined in the form of numerical values is accepted in mathematical methods. However, imprecise or incomplete information about a problem is often available in engineering calculations. For example, imprecise information concerning the material properties or strain is often knowable in problems of materials forming. The fuzzy logic theory might be used in situations for which precise data are not available [1].

Fuzzy logic is a superset of Boolean logic which introduces the term of partial truth. Whereas, the classical logic uses only binary terms (0 or 1), fuzzy logic implements the degrees of truth instead classical truth values [2]. The first mention about fuzzy logic theory was presented by Plato who postulated the existence of area between the truth and falsity [3]. But Lofti A. Zadeh is considered as a father of fuzzy logic theory. He indicated that fuzzy theory might be used for solving both easy and complicated problems. In one of his papers titled „Fuzzy

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Sets” published in 1965, he stated that *As the complexity of system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics* [4].

The fuzzy logic is applied in problems for which the knowledge of the mathematical model is unknown or model reconstruction is impossible or unprofitable. The fuzzy theory is necessary to describe unspecified phenomena which cannot be described with the use of classical logic. It enables to find the system response to external factors assuming that input data are fuzzed and the adopted mathematical model is uncertain. The fuzzy theory also enables to find the most reliable solution from obtained results [5].

Fuzzy logic was first implemented in England in 1973 at University of London. The aforementioned theory was used for the stabilization the speed of a small steam engine constructed for the research purposes [6]. Nowadays, fuzzy logic has become a design technology in many developed countries, such as Japan, Germany, Korea, Sweden and France [7]. The fuzzy theory is applied in controlling, databases, electronics, medicine, mechanical science, ecology and other branches of industry. One of the most popular implementations of fuzzy logic on a big scale is the subway in Sendai (Japan). The subway system uses a fuzzy logic system to control acceleration and thanks to it, journey is more comfortable with smooth braking and acceleration. The fuzzy logic controller is also responsible for traffic planning and the usage of subway facilities by passengers [8]. Fuzzy logic controllers are also used in anti-lock braking system (ABS) which affects the improvement of the braking performance and the directional stability of cars [9]. But the examples of fuzzy logic are also observed in everyday life. Application examples of fuzzy logic are washing machines and fuzzy microwaves. The application of fuzzy logic technology enables to choose the best washing cycle or the most appropriate temperature and time of cooking [10, 11].

Fuzzy logic is also used in scientific problems which were confirmed in research carried out by different researchers. Amjad et al. [12] tested the application of fuzzy theory to control the position of a ball in a ball and beam system. They showed that the fuzzy controller stabilized the system efficiently. Moreover, the performance during the transient period of the fuzzy system was better and less overshoot was obtained. The fuzzy controller also provided a zero steady state error [12]. Parthiban et al. [13] used fuzzy theory in order to predict the CO₂ laser cutting parameters. They noted that the proposed fuzzy model might be applied in prediction of the top kerf width, bottom kerf width and kerf deviation on stainless steel sheet. In other research, Giorleo et al. [14] examined the possibility of the application of fuzzy logic in the control of steel rod quenching after hot rolling. They showed that this concept might be helpful in the assessment of the influence of steel diameter and chemical composition on its final mechanical properties. All aforementioned examples indicate the usefulness of fuzzy logic in engineering application.

The possibility of the use of fuzzy logic in engineering problems is presented in this paper. The theoretical information concerning the fuzzy theory and the mathematical calculations on fuzzy sets is shown. Two methods used in fuzzy theory: extension principle and α -level optimization are described with an indication of their advantages and disadvantages. The way to choose the most reliable solution which is based on the selection of mass center and level rank method is also presented.

2. Theoretical basis of fuzzy logic theory

In order to consider the application of fuzzy logic in engineering applications, the definition of basic terms associated with fuzzy theory, as well as, the determination of mathematical operations on fuzzy sets are necessary. In classical theory, the x element of X space might belong or not belong to the set. The membership of x element to the set X might be 1 when the element belongs to the set or 0 when the element does not belong to the set [15]. The fuzzy logic implements the partial membership of x element to the set (Fig. 1a). The fuzzy set A is defined as a set of x elements of X space which indicates the some membership to the set (1) [16]. In this case, the X set is a space on which the fuzzy set A is determined (Fig. 1b).

$$A = \{(x, \mu_A(x)) : x \in X\} \quad (1)$$

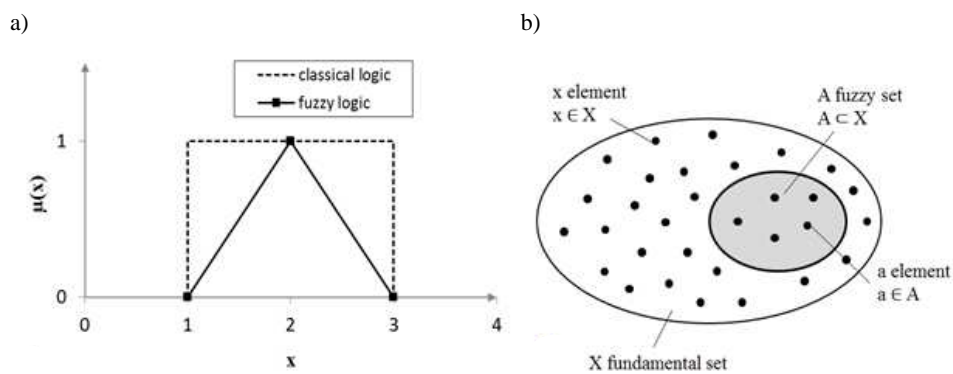


Fig. 1. Differences between classical and fuzzy logic (a) and the placement of A fuzzy set in X fundamental set (b)

The membership of x element into A fuzzy set is aimed by the membership function $\mu(x)$. For all x elements, this function assigns its membership to A fuzzy set (2). In other words, the membership function determines the level of truth that a variable has specific value. Its value is within the range from 0 to 1.

$$\mu_A : x \rightarrow [0,1] \quad (2)$$

The membership function is created on the basis of measurements, possible measurement errors, researcher's experience, etc. For this reason, different shapes of membership function are used. Triangular, trapezoidal, Gaussian as well as pseudo-Gaussian distributions are commonly acceptable shapes of the membership function [16]. Depending on the shape, it is determined with the use of some points belonging to the function. For example, triangle membership function is defined by means of three points which are its vertices. The normal Gaussian distribution is adequate for the most of real problem. But in practice, the linear distribution is most often used [17].

Main calculations on fuzzy sets include addition (Fig. 2a) and multiplication (Fig. 2b). For two fuzzy sets A_1 and A_2 of fundamental sets X_1 and X_2 , addition and multiplication might be expressed by the formulas (3) and (4):

$$A \cup B = \min(\mu_A(x), \mu_B(x)) \text{ for } x \in X \quad (3)$$

$$A \cap B = \max(\mu_A(x), \mu_B(x)) \text{ for } x \in X \quad (4)$$

In mapping step, the Cartesian product (K) is used. It includes all combinations of x elements of fuzzy sets A_1, \dots, A_n . With the application of minimum operator, membership function values $\mu_K(x) = \mu_K(x_1, \dots, x_n)$ are assigned to one n -tuple (5). The Cartesian product represents a fuzzy set in the product space X with the membership function $\mu_K(z)$ [16]. The extension principle which is used in fuzzy logic is based on the Cartesian product of input variables.

$$K = \{(x = (x_1, \dots, x_n), \mu_K(x) = \mu_K(x_1, \dots, x_n)) : x_i \in X; \mu_K = \min[\mu_{A_i}(x_i)]\} \quad (5)$$

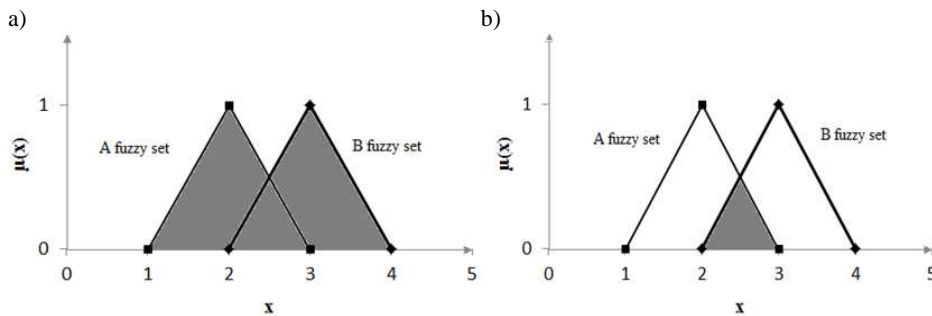


Fig. 2. Result of addition (a) and multiplication (b) of fuzzy sets

The aim of fuzzy logic theory is modeling of systems with a known mapping from input to output. The main assumption is imprecise defined input parameters. The membership function for the obtained result $z = f(x_1, x_2)$ is determined on the basis of calculations on fuzzy sets. The membership function also presents the possible dispersion of z variable, as well as, the level of accessibility of its

values [9]. The most reliable solution of z variable might be also determined in the defuzzification step (Fig. 3).

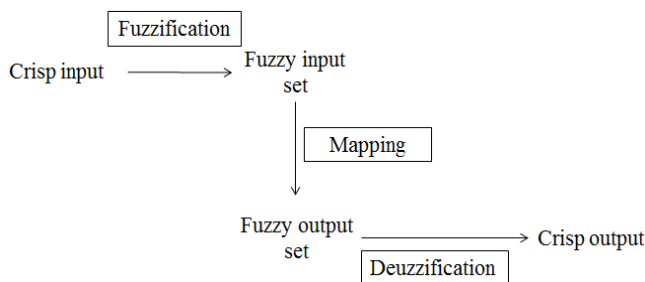


Fig. 3. Mapping of fuzzy input variables into output space

Two methods – extension principle and α -level optimization are used in fuzzy theory. The extension principle (Fig. 4) is based on the Cartesian product of fuzzy sets A_1, \dots, A_n . The final product of this method presents the fuzzy set with n -dimension membership function $\mu_K(x_1, \dots, x_n)$. It is an input fuzzy set. The extension principle presents the mapping of input set X on the new Z fundamental set with the use of mapping function $z = f(x_1, \dots, x_n)$ [11]. The new B fuzzy set and its membership function $\mu_B(z)$ is gained on Z set (6). The values of $\mu_B(z)$ are calculated with the use of max-min operator (7).

$$B = \{ (z, \mu_B(z)) : z = f(x_1, \dots, x_n); z \in Z; (x_1, \dots, x_n) \in X_1, \dots, X_n \} \tag{6}$$

$$\mu_B(z) = \{ \sup_{f=(x_1, \dots, x_n)} \min[\mu(x_1), \dots, \mu(x_n)], \text{ if } \exists z = f(x_1, \dots, x_n) \} \tag{7}$$

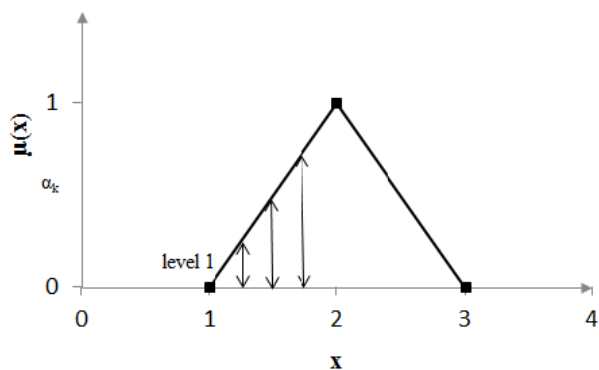


Fig. 4. Determination of membership function with the use of extension principle

In some special mappings, for example during addition or multiplication of fuzzy numbers, different shapes of membership function $\mu(z)$ might be achieved (Fig. 5). The addition of fuzzy triangular numbers $x = \langle x_1, x_2, x_3 \rangle$ and $y = \langle y_1, y_2, y_3 \rangle$ gives the triangular number $z = \langle z_1, z_2, z_3 \rangle$. The multiplication of x and y gives a fuzzy results with a nonlinear membership function [16].

The alternative method in fuzzy logic is α -optimization which is based on the discretization of support. This concept relies on the selection of sufficiently high number of α -levels (Fig. 6). The extreme $x_{\alpha_{kl}}$ and $x_{\alpha_{kr}}$ values determine the subspace assigned to α_k level. The $x_{\alpha_{k,l}}$ and $x_{\alpha_{k,r}}$ values for sufficiently high number of α -levels designate the shape of membership function [5, 16, 17].

For all α -level representation of fuzzy input variables, the $z_{\alpha_{kl}}$ and maximum $z_{\alpha_{kr}}$ variables are found. This searching is formulated as an optimization problem in which $(x, y) \in X_{\alpha_k}$ are constraints of the optimization process (8). Extreme values of z variable for each α -level determine $\mu(z)$ membership function. The result obtained in α -optimization method is smoother than envelop for extension principle [5]. But the α -optimization might be used if mapping operator is continuous and fuzzy result space is convex.

$$\begin{aligned} z &= f(x, y) \Rightarrow \max (x, y) \in X_{\alpha_k} \\ z &= f(x, y) \Rightarrow \min (x, y) \in X_{\alpha_k} \end{aligned} \quad (8)$$

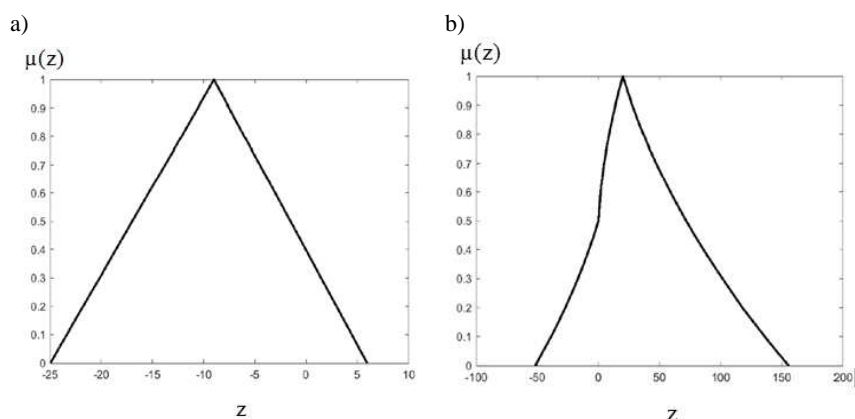


Fig. 5. Addition (a) and multiplication (b) of fuzzy variables

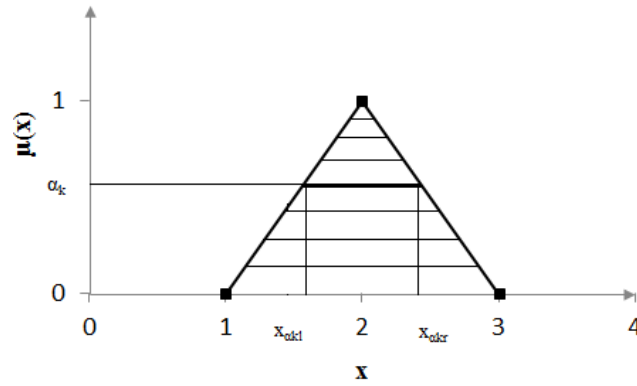


Fig. 6. Determination of membership function with the use of α -optimization method

As the final step of calculations on fuzzy sets, defuzzification of Z fuzzy variable on the non-fuzzy space is applied. It leads to the obtaining of specific value z_{j0} which is the most reliable solution. The mass center method is commonly used for the conversion of fuzzy variable into the non-fuzzy value. The center of space below the membership function plot is searched as the most reliable value (9) [16].

$$z_{j0} = \frac{\int_{z_j} z_j \cdot \mu(z_j) dz_j}{\int_{z_j} \mu(z_j) dz_j} \quad (9)$$

In order to achieve the most reliable solution, level rank method which is based on the α -discretization might be also used. The membership scale of z fuzzy variable is discretized with the aid of r chosen α -levels. After that, the arithmetic mean of the interval centers of α -level sets is calculated (10). The results obtained by means of this method might vary depending on the number of α -levels [16].

$$z_{j0} = \frac{1}{r} \cdot \sum_{k=1}^r \frac{z_{j,\alpha_{kl}} + z_{j,\alpha_{kr}}}{2} \quad (10)$$

3. Determination of membership functions for output variable with the use of extension principle and α -optimization

The determination of the membership function for z fuzzy variables with the use of two methods described in Section 2 is presented below. In considerations, the mapping model which is described as a function $z = f(x, y) = -x \cdot y + 50 \cdot \sin(x) + 40 \cdot \sin(y)$ was used. The three-dimensional and contour plots are presented in Figure 7. All numerical calculations were done with the use of MATLAB program.

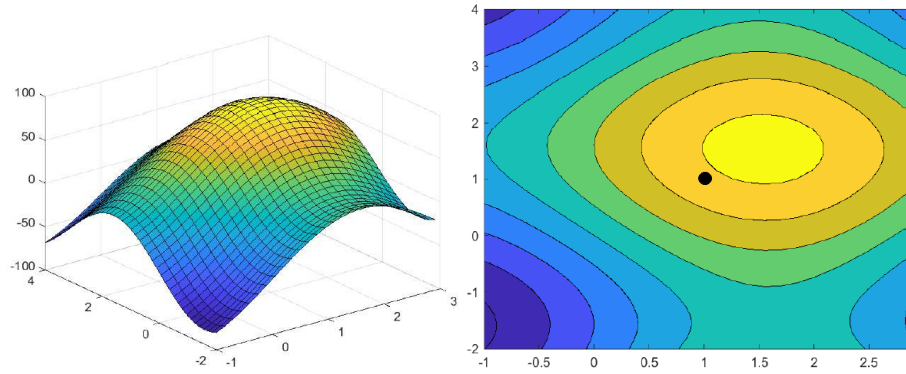


Fig. 7. Plot of z mapping function

It was assumed that arguments of x and y functions are fuzzy variables. The membership functions for x and y variables are presented in Fig. 8. On the basis of plots it was stated that $x \in [-1; 3]$ and $y \in [-2; 4]$. The most reliable value of x and y variables is 1. For these arguments, the value of membership functions is 1. Using membership functions for x and y , z result variable and its membership function were determined.

In extension principle, the membership function for z variable is obtained by means of random searching of range of arguments variability. The membership function is calculated with the use of minimum and maximum operator according to the following formula (11) [17].

$$\mu(z) = \sup \min[\mu(x), \mu(y)], \exists z = f(x, y) \quad (11)$$

The application of two operators is associated with the fact that there are many ways of mapping of X and Y fuzzy sets on Z result set. Different combinations of x and y elements gives different $\mu(z)$ values. Firstly, the values of membership functions $\mu(z)$ are determined with the use of minimum operator. After that, the highest $\mu(z)$ value for z element is selected. The membership functions for ranges of fuzzy variable divided into $n = 500$ and 5000 intervals are presented in Figure 9. The thick line represents the sup operator.

Although the membership functions for x and y input variables might be linear (triangular and trapezoidal), the membership function for z output variable is always non-linear. It was observed that the small amount of n subdomains gives a jagged solution. Better results might be achieved for higher number of analyzed subdomains. The results presented in Fig. 9 show that the extension principle is very sensitive to the number of combinations of x and y elements of fuzzy input data. The final result is also dependent on the assumed precision Δz of searching maximum from minimum values of membership function. Additionally, a smooth envelope might be obtained only for dense searching of the variability range of

fuzzy input data [5]. It is related with the multiple calculation of mapping function values.

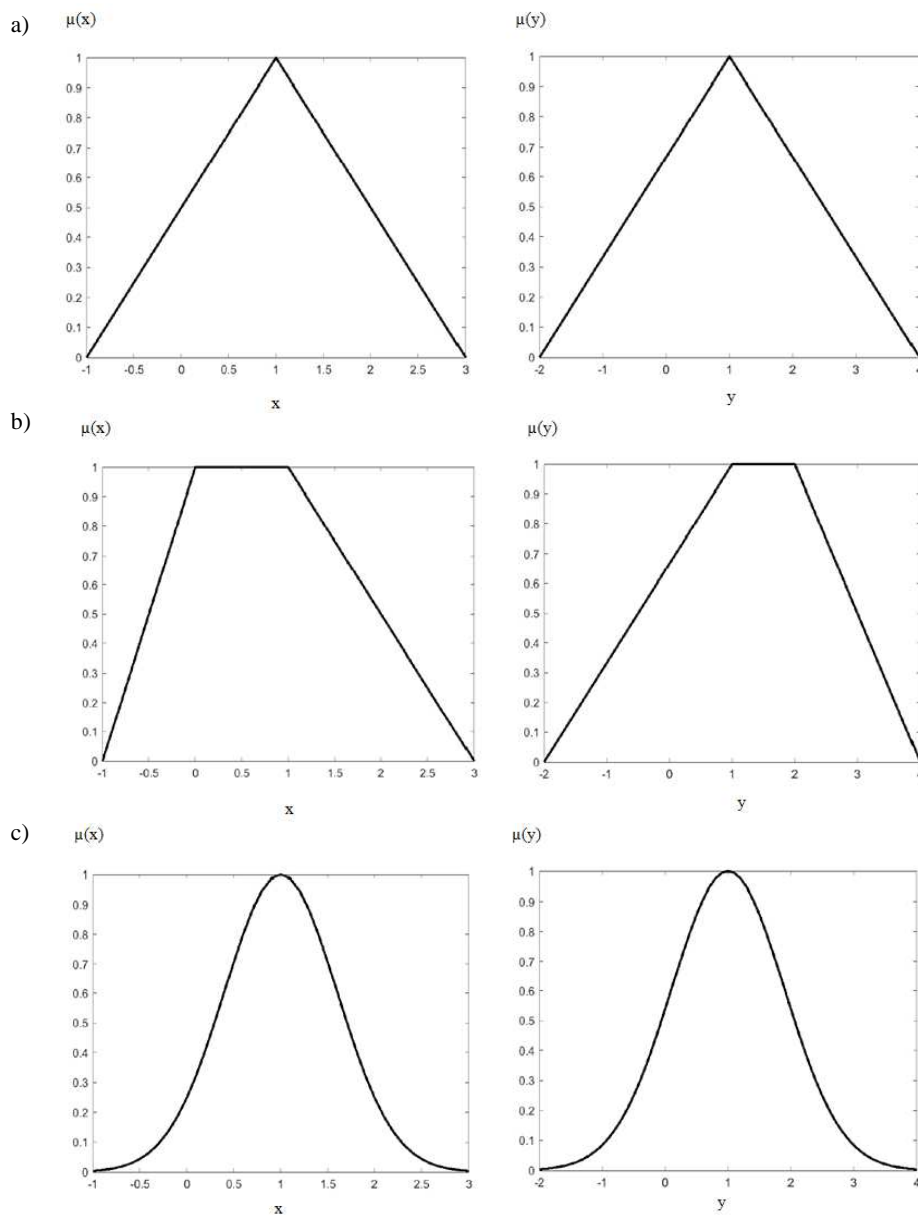


Fig. 8. Membership functions for x and y variables: triangle (a), trapezoidal (b) and pseudo-Gaussian (c)

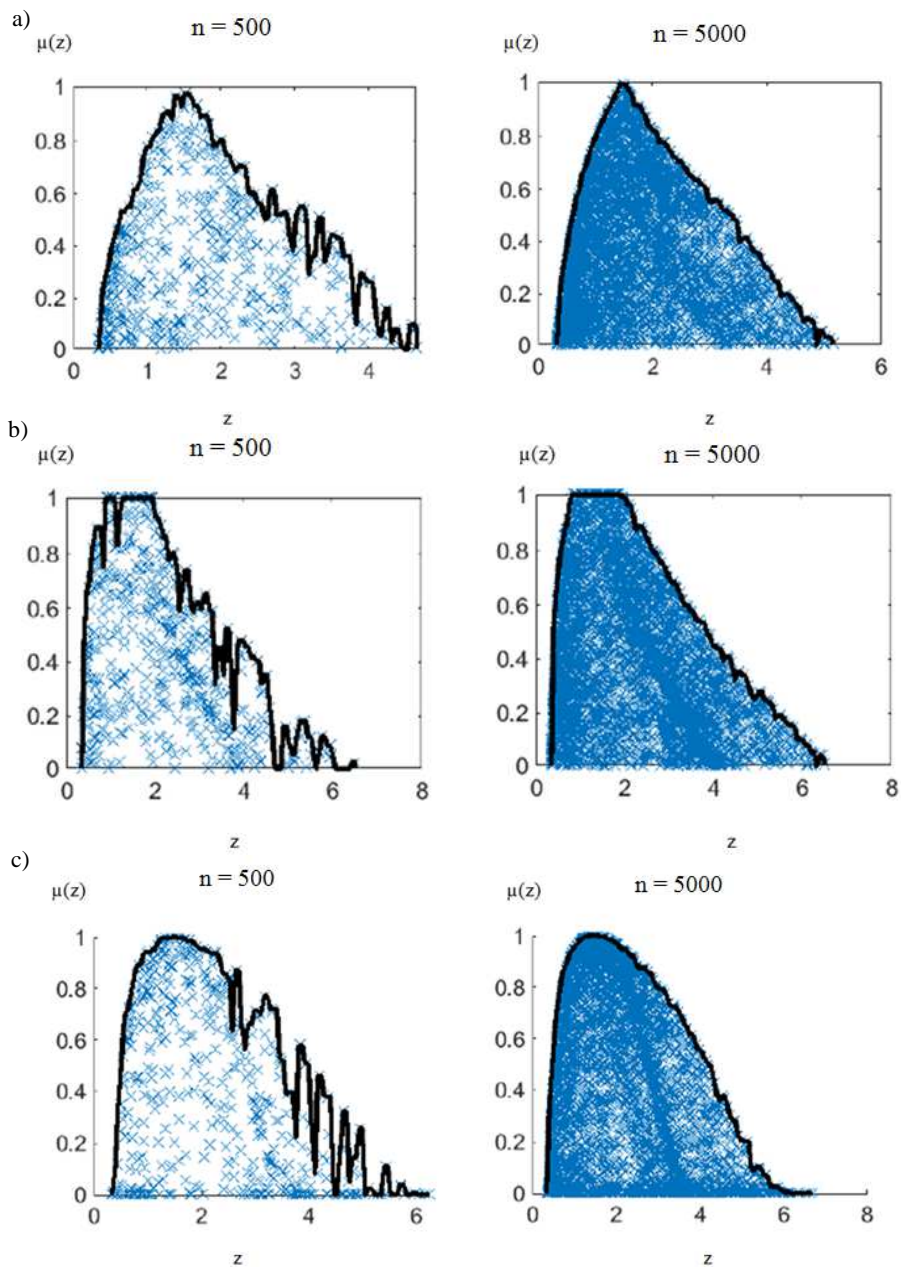


Fig. 9. Results obtained for $n = 500$ and $n = 5000$ subdomains for triangle (a), trapezoidal (b) and pseudo-Gaussian (c) input membership functions

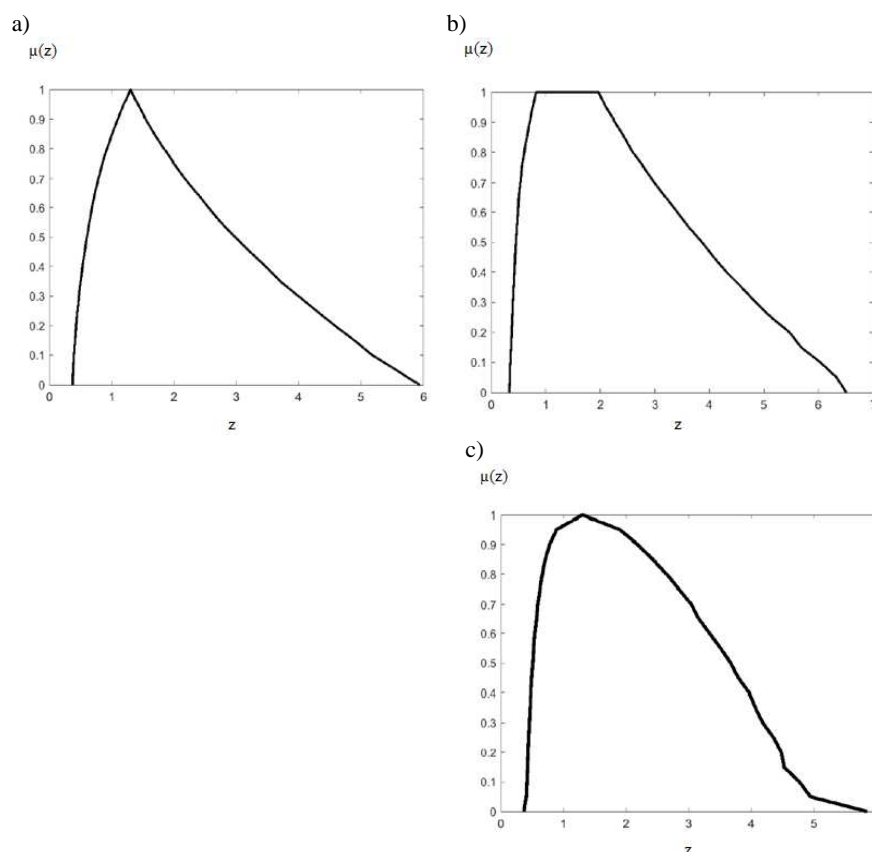


Fig. 10. Fuzzy value of z function obtained with the use of α -optimization for triangle (a), trapezoidal (b) and pseudo-Gaussian (c) input membership functions

The extension principle is ineffective for a mapping model which has a form of complex numeric algorithm. In this situation, α -optimization method is more suitable. This concept enables to decrease the number of calculations for fuzzy output variable z . Firstly, the membership function is divided into the high number of α -levels. For all considered α -levels, $x_{\alpha_{kl}}$ and $x_{\alpha_{kr}}$ values for which $\mu(x) = \alpha_k$, are selected. These points define the shape of membership function. After that, the minimum $z_{\alpha_{kl}}$ and the maximum $z_{\alpha_{kr}}$ elements are searched with the use of mapping operator. Extreme z values for all α -levels define the membership function $\mu(z)$. Detailed information concerning the α -optimization method was presented in Section 2.

The result obtained with the use of α -optimization is presented in Fig. 10. The membership value is 1 for $z = 1.30$. In comparison to result for extension principle, α -optimization method enables to achieve definitely smoother solution. In extension principle, such smooth line result might be attained only for very

dense discretization of input fuzzy variables. For this reason, α -level optimization procedure is recommended in structural analysis [5, 17].

Defuzzification of z variable on the non-fuzzy space is investigated with the use of mass center method. The most reliable value obtained with the use of mass center method is 1.53; 1.54 and 1.52 for triangle, trapezoidal and pseudo-Gauss input membership functions, respectively. This value is similar for all shapes of input membership functions. It shows that the shape of membership function does not affect final result. The most reliable value which was achieved with the use of rank level method is 1.52; 1.56 and 1.55 for triangle, trapezoidal and pseudo-Gauss input membership functions, respectively. On the basis of obtained results it can be stated that values achieved with the use of two aforementioned methods are similar.

4. Summary

Fuzzy logic is a relatively new concept in science which might be used for the analysis of mathematical models contained some uncertainties. Because many problems in technical sciences are more or less uncertain, the application of classical logic might be impossible. The fuzzy logic can be used in problems for which mathematical model is unknown. The fuzzy theory also investigates the influence of selected parameters on the model response.

The aim of fuzzy logic is modeling of systems with a known mapping from input to output. The main assumption is fuzziness of input parameters. Two methods: extension principle and α -optimization might be used in order to investigate the membership function for the output variable.

Based on the results obtained with the use of MATLAB program, the following findings and conclusions could be made:

- In the extension principle, more calculations in order to achieve satisfactory results are necessary. The small amount of calculations gives a jagged solution. Smooth envelope might be obtained only for dense searching of the variability range of fuzzy input data.
- The extension principle is very sensitive to the number of combinations of x and y elements of fuzzy input data. Additionally, the final result is also dependent on the assumed precision Δz of searching maximum from minimum values of membership function.
- The α -optimization method is more suitable for more complicated engineering problems. This method enables to decrease the number of calculations for fuzzy output variable.
- It was noted that the shape of input membership function does not affect significantly on the most reliable value. This value is similar for triangle, trapezoidal and pseudo-Gauss input membership functions.
- The most reliable values which are obtained with the use of mass center and rank level method are similar.

- There are differences between the value for which membership function is 1 and the value measured with the use of rank level or mass center methods. The differences are associated with the non-linear character of mapping model.

The fuzzy theory indicates the potential application in solving engineering problems and gives methodical flexibility to engineers on calculations. Its application on a big scale enables to optimize most of technical processes. In this article, fuzzy logic was used for solving simply problem. In further research, fuzzy theory will be applied for specific engineering problems. Due to the lower number of necessary calculations, α -optimization method will be used. The extension principle will be applied in order to check the consistency of results.

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ZASTOSOWANIE LOGIKI ROZMYTEJ W INŻYNIERII MECHANICZNEJ

Streszczenie

Do opisu zjawisk, w przypadku których dane wejściowe lub model matematyczny nie są dokładnie znane, zastosowano logikę rozmytą. Teoria rozmyta umożliwia znalezienie najbardziej wiarygodnego rozwiązania przy założeniu rozmycia danych wejściowych. Artykuł przedstawia możliwości zastosowania teorii rozmytej w inżynierii mechanicznej. Zaprezentowano teoretyczne podstawy logiki rozmytej oraz opisano podstawy obliczeń matematycznych na zbiorach rozmytych. Opisano i porównano ze sobą dwie metody stosowane w teorii rozmytej: α -optymalizację oraz zasadę rozszerzeń. W artykule przedstawiono przykłady zastosowania tych metod do rozwiązania prostego problemu inżynierskiego. Wszystkie obliczenia numeryczne wykonano z użyciem programu MATLAB. Przedstawiono również metodę wyboru najbardziej wiarygodnego rozwiązania opartego na poszukiwaniu środka ciężkości figury ograniczonej wykresem funkcji przynależności oraz na wyznaczeniu średniej arytmetycznej ze środków przyjętych poziomów α .

Słowa kluczowe: logika rozmyta, α -optymalizacja, zasada rozszerzeń, inżynieria mechaniczna

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