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THEORETICAL BASIS OF THE LASER REFLECTION METHOD OF MEASURING THE CIRCULAR-SYMMETRICAL LENSES CURVATURE

In this paper, a theoretical analysis of the laser reflection method for measurement of the radius of curvature of circular-symmetric lenses was carried out. The measurements of the radius of curvature of spherical and aspherical lenses were taken. The use of the criterion function in the calculation for minimalization of the uncertainty of radius curvature for a spherical lens has been demonstrated. The parameters of the function describing measurements and minimizing their uncertainty were determined.

Keywords: spherical lens, aspherical lens, radius of curvature, criterion function

1. INTRODUCTION

In testing transparent elements with curved surfaces, including circularly symmetric ones, the radius of curvature of spherical lenses is measured with a spherometer [6] or using the interference method with Newton's fringes for small curvatures [1].

The value of the radius of the lens enables to determinate its parameters, for example the focal length and optical power [8,11]. Contact methods can only be used for lenses made of glass or polymer. However, for liquid lenses [2 ,5] it is necessary to measure the radius of curvature using the non-contact method. Aspherical lenses are also used as fiber optic terminations in endoscopic imaging [9, 10]. The technology of making lenses has already been developed to such an extent that the curvature of the lens can be controlled during the production process. The quality of the lens is checked, for example, using X-ray micro tomography and atomic force microscopy [9].

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The work presents an indirect method of measuring the radius of curvature of two types of lenses, spherical and aspherical, using scanning of their surface with a laser beam. The position on the screen of the trace of its beam reflected from the surface of this lens is measured. An equation containing the radius of curvature is non-linear. The values of its parameters are determined after its linearization by changing variables and fitting to the measurement results for the minimum of the WTLS (Weighted Total Least Squares) criterion function as the sum of the weighted squares of the residual variable and measurement uncertainty. This method for nonlinear functions, also including those with an implicit form, was developed by Puchalski and Warsza [12, 13]. Some examples of using this method in laser optical measurements systems were given in briefly for the first time in presentations at three scientific conferences in June 2024: at the XV Measurement Systems in Research and Industry SP´2024 in Łagów, Poland [14], and at the Problems and Progress in Metrology PPM'24 in Gliwice, Poland and later in September of the same year at the Inter-University Conference of Metrologists 56MKM in Kalisz, Poland [15]. The first version is published in Przegląd Elektrotechniczny [16] in Polish, and in English in open access journal, available at Metrology of MDPI [17]. The work mentioned is the extended version only about application of the new method of fitting nonlinear function in laser optical systems.

2. MEASUREMENTS OF THE RADIUS OF CURVATURE OF A SPHERICAL LENS

In this part an example of a laser method for measuring the radius of curvature will be discussed. It is based on the examination of the position of the trace of a laser beam reflected from the surface of the lens. The measurement system is shown in Figure 1.

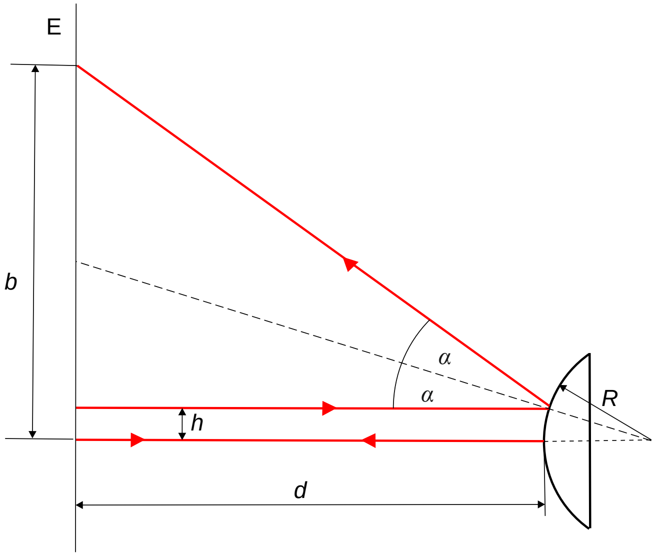

Fig. 1. A diagram of the system for measuring the radius *R* of spherical lenses

The lens is placed centrally to the laser beam, at a distance of *d* from the screen E. Uncertainties of the parameters will be calculated here in accordance with recommendations of Guide GUM [4], named by Zięba [7] as the GUM convention. Uncertainty of the distance *d* is $u(d) = 0.1\mu m \approx 0$. The laser beam falls on the lens surface at a distance *h* from its optical axis. Uncertainty $u(h) = 0.001$ mm. After reflecting from the surface of the lens, the beam makes a luminous spot on the screen at a distance *b* from its axis. A series of measurements of the distance *b* are made with uncertainty $u(b) = 0.1$ mm, for different values of *h*. Based on the results of these measurements, the radius *R* is determined. For α as the angle of incidence and the reflection of the laser beam, a basic geometric relationship is obtained:

$$\frac{b-h}{tg2\alpha} = d + R(1-\cos\alpha) \tag{1}$$

The description of the measurement system includes the following trigonometric relationships:

$$tg(2\alpha) = \frac{2tg\alpha}{1-tg^2\alpha} = \frac{2sin\alpha\sqrt{1-sin^2\alpha}}{1-2\,sin^2\alpha},$$

$$\frac{h}{R} = sin\alpha, \quad \frac{dh}{d\alpha} = R\cos\alpha \tag{2a – c}$$

From equation (1) the following can be obtained:

$$\frac{b}{tg2\alpha} = d + R\left(1 - \frac{1}{2cos\alpha}\right) \tag{3}$$

This is a non-linear equation with respect to the radius R, as the values of the angle α depend on R. The value of R is determined after adjusting the parameters of the equation describing the measurements to the measurement results using a method that uses the linearization of a non-linear function by changing variables. For this purpose, new variables are introduced:

$$z(b,\frac{h}{R}) = \frac{b}{tg2\alpha} = b\frac{1 - 2\,sin^2\alpha}{2sin\alpha\sqrt{1 - sin^2\alpha}} = b\frac{1 - \frac{2h^2}{R^2}}{\frac{2h}{R}\sqrt{1 - \frac{h^2}{R^2}}} \tag{4}$$

and

$$\xi\left(\frac{h}{R}\right) = 1 - \frac{1}{2cos\alpha} = 1 - \frac{1}{2\sqrt{1 - \frac{h^2}{R^2}}} \tag{5}$$

Equation (3) for these variables becomes linear:

$$z = R\xi + d \tag{6}$$

Moreover, it is assumed that the distance d of the screen E from the tested lens is constant throughout the measurement process, and that the influence of cross-correlation and autocorrelation of the measurement results is negligible. After changing the variables $h \longrightarrow \xi$ and $b \longrightarrow z$, for different distances h_i of the laser beam from the lens axis, b_i_ measurements are made. The coordinates of the measured points have the values ξ_i and z_i and the uncertainties $u(\xi_i)$ and $u(z_i)$. From the measurement results for each h_i, the R_{pi} values of the lens radius are determined, minimizing the WTLS criterion function $\phi_{\xi z}(\boldsymbol{\Delta\xi}, \boldsymbol{\Delta z}) \approx \phi_{h,b}(\boldsymbol{\Delta h}, \boldsymbol{\Delta b})$ [7-9], described by the formula:

$$\phi_{\xi z}(\boldsymbol{\Delta\xi}, \boldsymbol{\Delta z}) = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \left(\frac{\Delta\xi_i}{u(\xi_i)}\right)^2 + \left(\frac{\Delta z_i}{u(z_i)}\right)^2 \tag{7}$$

where the components of the error vectors $\boldsymbol{\Delta\xi}$ and $\boldsymbol{\Delta z}$ after changing the variables (the error vectors before the change are denoted by $\boldsymbol{\Delta h}$ and $\boldsymbol{\Delta b}$), are: $\Delta\xi_i = \xi_{pi} - \xi_i$ and $\Delta z_i = z_{pi} - z_i$, for $i = 1, \ldots, n$.

Selection of points with coordinates ξ_{pi} and $z_{pi} = R\xi_{pi} + d$ lying on the line described by equation (6) and minimizing the criterion function according to the condition ξ_{pi} and $z_{pi} = R\xi_{pi} + d$ is carried out analytically, obtaining the so-called effective covariance matrix, in this case a diagonal matrix, whose elements are standard effective uncertainties given by:

$$u_{eff}^2(\xi_i) = u^2(z_i) + R^2 u^2(\xi_i) \tag{8}$$

The criterion function (7), minimized locally by the choice of coordinates ξ_{pi}, and the condition $\frac{\partial\phi_{\xi z}}{\partial d} = 0$ (see equation (9g), is a quasi-quadratic function dependent on R:

$$\phi_{\xi z}(R) = R^{\mathbf{2}}\left(S_{\xi\xi} - \frac{S_\xi^2}{S}\right) + 2R\left(\frac{S_\xi S_{\boldsymbol{z}}}{S} - S_{\xi z}\right) + S_{zz} - \frac{S_z^2}{S} \tag{9}$$

Its auxiliary parameters are (9a - f), and the distance of beam d from lens axis is (9g)

$$S = \sum_{i=1}^{n} \frac{1}{u_{eff}^2(\xi_i)}, \quad S_\xi = \sum_{i=1}^{n} \frac{\xi_i}{u_{eff}^2(\xi_i)},$$

$$S_{\xi\xi} = \sum_{i=1}^{n} \frac{\xi_i^2}{u_{eff}^2(\xi_i)}, \quad S_z = \sum_{i=1}^{n} \frac{z_i}{u_{eff}^2(\xi_i)},$$

$$S_{\xi z} = \sum_{i=1}^{n} \frac{\xi_i z_i}{u_{eff}^2(\xi_i)}, \qquad S_{zz} = \sum_{i=1}^{n} \frac{z_i^2}{u_{eff}^2(\xi_i)}, \quad d = (S_z - RS_\xi)/S$$

(9a-g)

The measurement result of the lens radius R is therefore obtained from the WTLS criterion for the global minimum of the criterion function $\phi_{\xi z}(\boldsymbol{\Delta\xi}, \boldsymbol{\Delta z})$. Obtaining this minimum is illustrated in Figure 2a.

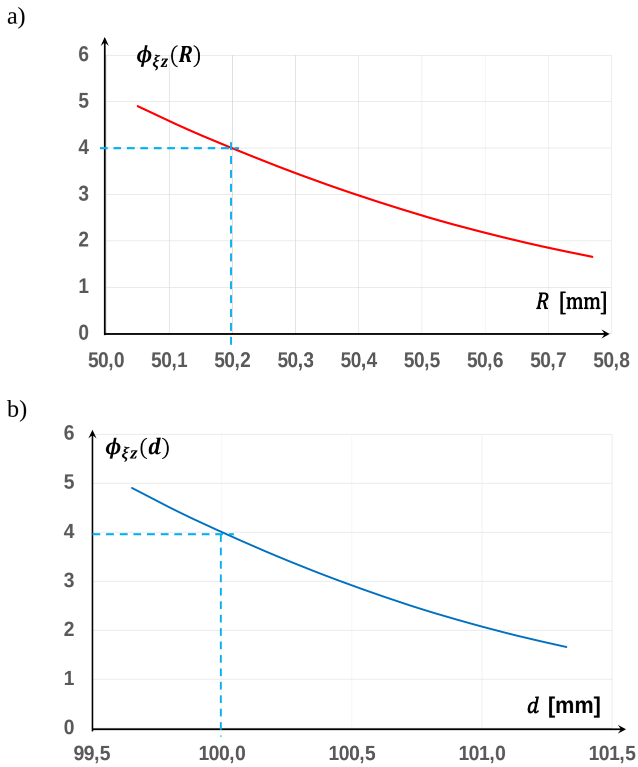

Fig. 2. Plots of the criterion function for a local minimum obtained for the effective uncertainty: a) dependence on the radius *R* of the lens, b) dependence on the distance of the lens from screen *d*

The uncertainties of both new variables $u(\xi)$ and $u(z)$ are described by the formulas

$$u(\xi) = \frac{1}{2}\frac{\frac{h}{R^2}}{\left(1 - \frac{h^2}{R^2}\right)^{\frac{3}{2}}} u(h),$$

$$u(z) \approx \frac{1 - \frac{2h^2}{R^2}}{\frac{2h}{R}\sqrt{1 - \frac{h^2}{R^2}}} u(b) \qquad (10\text{a, b})$$

These uncertainties depend on *R*. The calculations show that the uncertainty $u(z)$ is dominated by uncertainty $u(b)$, i.e. $u(z) \sim u(b)$. The uncertainty $u(\xi)$ depends on $u(h)$, while $u(z)$ practically does not depend on $u(h)$. After obtaining a match that minimizes the criterion function, the uncertainty of the radius R is determined from the uncertainty propagation law. It can be assumed that there is no correlation between the variables ξ and z (the correlation coefficient resulting from the common measurement variable h is below 0.1).

The value of the standard uncertainty of the radius R is then determined from the uncertainty of standard measurements b and h and from the numerically determined sensitivity coefficients, i.e. numerically estimated with the difference quotients of the first partial derivatives $\partial R/\partial h_i$ and $\partial R/\partial b_i$ (i =1, ..., n) at the point the adopted global minimum of the criterion function. Uncertainty is calculated from the following formula:

$$u^2(R) = u^2(h)\sum_{i=1}^{n}\left(\frac{\partial R}{\partial h_i}\right)^2 + u^2(b)\sum_{i=1}^{n}\left(\frac{\partial R}{\partial b_i}\right)^2 \qquad (11)$$

Table 1 shows the results of the measurements of b_i positions of laser beam traces in the system from Fig.1, performed at various h_i distances of the laser light beam from the lens axis.

Table 1. Location of traces b of the reflected light beam on the screen from the lens falling on it at a distance h from its axis, d = 100 mm

i	1	2	3	4	5	6	7	8	9	10
h_i [mm]	4	5	6	7	8	9	10	11	12	13
α_i[deg]	4,59	5,73	6,89	8,03	9,19	10,34	11,05	12,68	13,84	15
b_i [mm]	20,2	25,3	30,6	35,9	41,4	47,0	52,9	58,9	65,1	71,6
R_i [mm]	49,94	50,14	50,04	50,09	50,15	50,21	50,13	50,18	50,26	50,31
$u(R_i)$ [mm]	1,24	0,99	0,82	0,70	0,61	0,54	0,48	0,43	0,39	0,36

The global minimum in the method used occurs for R_{global} approximately 51.30 mm for the value of the criterion function $\phi_{\xi z global\_\min} \approx 0{,}96$. However, the distance from the d_{global} screen would then have to be 102.7 mm. It exceeds the d_{max} value because the screen-lens distance in this measurement process is d = 100 mm. Figure 2b shows that for this screen distance the value of the criterion function is $\phi_{\xi z \min} \approx 4$. From Figure 2a the value of criterion function is approximately 4, and the radius of the lens R = 50.2 mm is obtained as an average from the measurements for various values of h.

Example 1 The second method of estimating the lens radius

Values of the lens radius R_i at each measurement point i =1, …, n and their standard uncertainties $u(R_i) = \frac{1}{\xi_i}\sqrt{u^2(z_i) + \frac{u^2(\xi_i)(z_i-d)^2}{\xi_i^2}}$ can be determined from the nonlinear implicit equation (6). For a spherical lens of constant radius R, on the figure 3 given are measured points in polar coordinates (R_i, α_i), and the function $R(\alpha) = \bar{R} =$ const. as a straight line is fitted to these points. Also, two uncertainty bands: standard *u* and the expanded one *U* =2 are also added to the function.

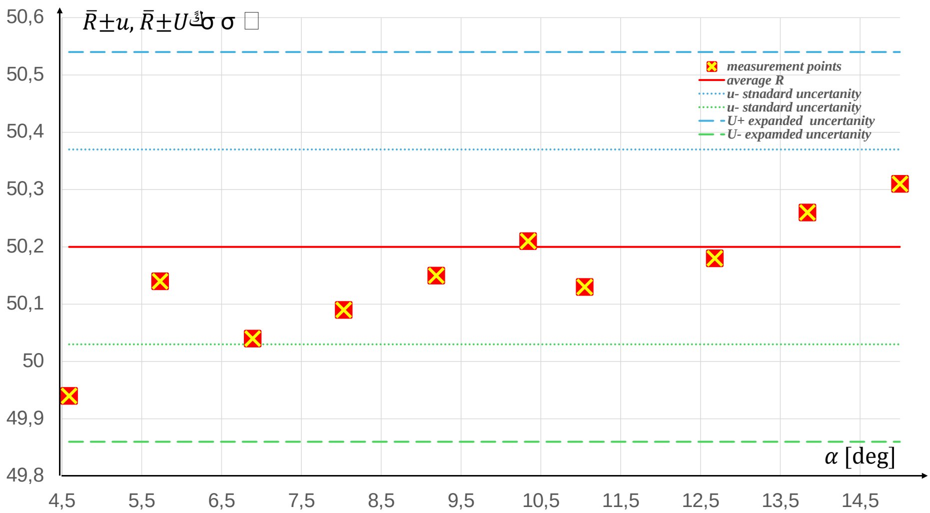

Fig. 3. The measured points of a spherical lens of radius *R* =cost and the function *R*=*f*(*α*) fitted to them as a straight-line $\bar{R}$ = 50.2 mm, and its two uncertainty bands: standard *u*(*α*), and extended *U*(*α*) =2*u*

The average value $\bar{R}$ of the radius of curvature and its standard uncertainty from weighted averaging variances for the measured number of points can be determined from the LS method. In order to determine *R* by the least squares method. The following formula is used:

$$\phi = \sum_{i=1}^{n} \frac{(R - R_i)^2}{u^2(R_i)} \rightarrow \quad min$$

Calculating the first derivative $\partial\phi/\partial R$ and setting it to zero, we obtain:

$$\frac{\partial\phi}{\partial\, R} = 2\sum_{i=1}^{n} \frac{R - R_i}{u^2(R_i)} = 0$$

Therefore,

$$\bar{R} = \sum_{i=1}^{n} \frac{R_i}{u^2(R_i)} \; / \sum_{i=1}^{n} \frac{1}{u^2(R_i)} = 50{,}2 \text{ mm}$$

On the other hand, the standard uncertainty is expressed from the law of propagation of uncertainty by the following formula:

$$u(\bar{R}) = 1/\sqrt{\sum_{i=1}^{n} \frac{1}{u^2(R_i)}} = 0{,}17 \text{ mm}$$

3. MEASUREMENTS OF THE RADIUS OF CURVATURE OF ASPHERICAL (PARABOLIC) LENSES

Figure 4 shows a diagram of the course of laser beams of the system for measuring the radius *R* of aspherical lenses. The measurements are performed in a similar way to the previous example, but for different values of *h*.

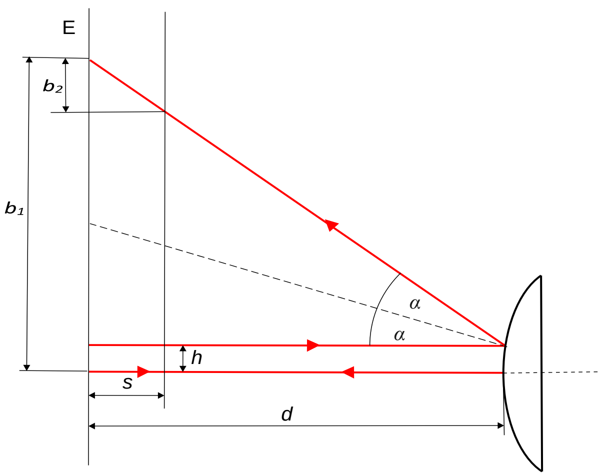

Fig. 4. A diagram of the system for measuring the radius *R* of aspherical lenses

For the set *h* value, the deviations b_1 and b_2 are measured and *x* is calculated according to the formula:

$$x = \frac{b_1 - h}{b_2} s - d \tag{12}$$

For $u(b_1) >> u(h)$, i.e. $u(b_1 - h) \approx u(b_1)$, the law of uncertainty propagation [6] results:

$$u^2(x) = \left(\frac{b_1 - h}{b_2}\right)^2 u^2(s) + \left(\frac{s}{b_2}\right)^2 u^2(b_1) + \left(\frac{(b_1 - h)s}{b_2^2}\right)^2 u^2(b_2) + u^2(d) \tag{12a}$$

Examples of h, b_1 and b_2 values for different values of x and $s = 2.900 \pm 0.001$ cm and $d = 9.800 \pm 0.001$ cm are given in Table 2. Additionally, $tg(2\alpha) = b_2/s$.

Table 2. Measured results and calculated values of the radius of curvature of aspherical lens

h[cm]	0,500	0,750	1,000	1,250	1,500	1,750	2,000	2,250	2,500	2,750	3,000
b_1[cm]	2,801	4,059	5,608	7,179	9,038	11,037	13,259	16,013	19,258	23,499	28,920
b_2[cm]	0,679	0,974	1,349	1,725	2,175	2,656	3,189	3,853	4,633	5,656	6,960
α_i[deg]	6,59	9,28	12,47	15,37	18,43	21,24	23,86	26,52	28,98	31,43	33,69
x [cm]	0,030	0,060	0,110	0,170	0,250	0,340	0,440	0,560	0,690	0,840	1,000
R [cm]	4,570	4,675	4,824	5,020	5,259	5,550	5,890	6,290	6,730	7,240	7,810
$u(x)$[cm]	0,151	0,106	0,077	0,060	0,048	0,040	0,034	0,028	0,024	0,020	0,017
$u(R)$[cm]	0,083	0,088	0,096	0,110	0,120	0,140	0,160	0,180	0,210	0,230	0,270
$U(R)$[cm]	0,166	0,177	0,192	0,213	0,239	0,271	0,308	0,352	0,402	0,460	0,526

Table 2 shows the values of the radius of curvature for different values of h. From the above data, the values of the radius of curvature R and its standard and extended uncertainties for the coverage probability $p = 0.95$ were calculated. The equation of a parabolic curve fitted to coordinates of measured data is:

$$h^2 = ax \quad \text{or} \quad x = h^2/a \tag{13}$$

It is adjusted to the results of measurements using the linearization method by changing variables [7-9]. For this purpose, the following is substituted: $g = h^2$ and uncertainty $u(g) = 2h\, u(h) \leq 0{,}0006\ \text{cm}^2$. Using the least squares method to approximate $u(g) \approx 0$, the values of a and $u(a)$ are determined [10] as:

$$a = \frac{W\, W_{gg} - W_g^2}{W W_{gx} - W_{gx}} = 8{,}971\ \text{cm},$$

$$u(a) = \frac{1}{a^2}\sqrt{\frac{W}{W\, W_{gg} - W_g^2}} = 0{,}313\ \text{cm} \tag{14a, b}$$

where

$$W = \sum_{i=1}^{n} \frac{1}{u^2(x_i)}, \quad W_g = \sum_{i=1}^{n} \frac{g(h_i)}{u^2(x_i)},$$

$$W_{gg} = \sum_{i=1}^{n} \frac{g^2(h_i)}{u^2(x_i)}, \quad W_x = \sum_{i=1}^{n} \frac{x_{i,}}{u^2(x_i)}, \qquad W_{gx} = \sum_{i=1}^{n} \frac{g(h_i)x_i}{u^2(x_i)}$$

(15a -e)

Figure 5 presents the values $R_i(\alpha_i)$ of the parabolic lens radius in polar coordinates and parabola function R=$f(a)$ obtained by the linear regression method.

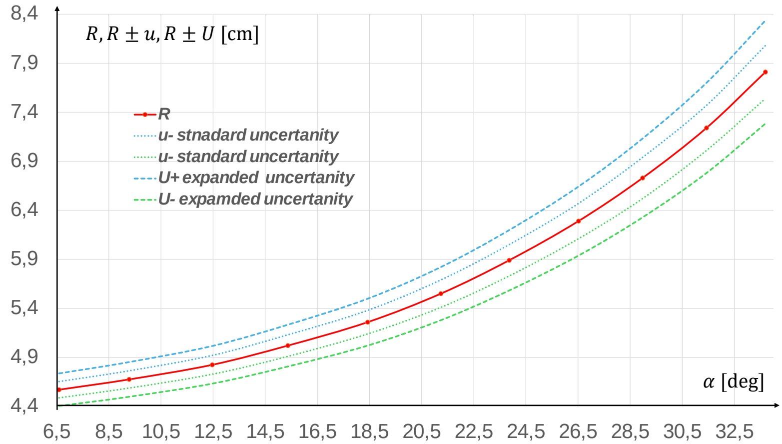

Fig. 5. The function $R=f(\alpha)$ of ideal aspherical lens as parabola fitted to the measured points and its two uncertainty bands: standard $u(\alpha)$ and extended one $U(\alpha)$ =2u

The radius of curvature R is described by the value of the function $h = 2{,}995 \cdot \sqrt{x}$ ($\sqrt{a} = 2{,}995$), resulting from the equation [10]:

$$R(h) = \frac{(1+h'^2)^{\frac{3}{2}}}{h''} = \frac{(4h^2+a^2)^{\frac{3}{2}}}{2a^2} \tag{16}$$

The formula for the relationship between the length of the radius R vs. the angle a is much simpler

$$R(a) = \frac{(a^2tg^2a + a^2)^{\frac{3}{2}}}{2a^2} = \frac{a}{2\cos^3\alpha} \tag{17}$$

because the perpendicular to the tangent (13) is inclined at the angle a and $tga = -(-1/(\frac{dh}{dx})$=$1/(\frac{1}{2}\sqrt{a/x}) = 2\sqrt{x/a} = 2h/a$. It is marked as a theoretical red curve in Figure 4.

For the different values of h, a different R value is obtained with different values of standard uncertainty h, according to the Gaussian distribution.

$$u^2(R) =$$

$$\frac{36h^2(4h^2+a^2)}{a^4}u^2(h) + \frac{1}{4}\left(\frac{3(4h^2+a^2)^{\frac{1}{2}}}{2a} - 2\frac{(4h^2+a^2)^{\frac{3}{2}}}{a^3}\right)^2 u^2(a) \tag{18}$$

The standard uncertainties $u(R)$, and extended uncertainties $U = k_p \cdot u(R)$ with the probability p=0.95 for k=2 are also shown in Table 2.

4. CONCLUSIONS

The described optical method is remote and contactless. It is suitable for measuring the curvature of spherical and aspherical lenses with anti-reflective layers and organic lenses with liquid.

In the analysis of the measurement results of a spherical lens, a new method of fitting a non-linear function to the data of measurement with linearization by changing variables was used [12] - [17]. This method can also be used to measure the radius of curvature of the lens during the accommodation process on the eye model.

The radius R of a spherical lens is obtained from a series of measurements for different distances h of the laser beam point of reflection from the lens axis.

The measurement method and its description were optimized using a criterion function. The value of the criterion function will be the same in this case. However, for an aspherical lens, each measured point has a different radius R, and the values of the criterion function will be different.

REFERENCES

[1] Meyer-Arendt J.R., Introduction to Classical and Modern Optics, Practice -Hall, Inc. Englewood Cliffs, N.J. 1972

[2] Buczaj M., Using Fluid Focus Liquid Lens System and THR Threshold Processing to Upgrade Gastroscopic Testing. (in Polish: *Wykorzystanie systemu soczewek cieczowych Fluid Focus i przetwarzania progowego THR do unowocześnienia badan gastroskopowyc*h) PAK (*Pomiary Automatyka Kontrola*) 3,2005 pp.37-40

[3] Polyanin A.D., Manzhirov A.V., Handbook of Mathematics for Engineers and Scientists, Chapman &Hall/CRC, London, 2007

[4] Guide to the Expression of Uncertainty in Measurement, BIPM, 2008 +Suplements

[5] Tsai F.S., Johnson D., Francis C.S., Cho S.H., Qiao W., Adriapour A., Mintz Y., Morgan S., Talmini M., Lo Y., Fluidic lens laparoscopic zoom camera for minimally invasive surgery, Journal of Biomedical Optics 15 (3) 2010

[6] Gross H., Dorband B., Muller H., Handbook of optical systems, Wiley-Vach Verlag GmbH 2012

[7] Zięba A., Analysis of Experimental Data in Science and Technology Cambridge Scholar Publishing 2023 ISBN (10): 1-5275-0448-4 ISBN (13): 978-1-5275-0448-6 (It is extended version in Polish: *Analiza danych w naukach ścisłych i technice*, Warszawa, PWN 2014)

[8] Milewski V., Chizh I., Testing Metods and hardware intraocular lenses. – in Ukrainian, Biulletin of Winnica Technology University. 2015 no 3. pp.7-14

[9] Karanukaran B., Tharion J.,T., Ramrad A.,R., Dhawangele D.,P., Mukherji S.,M., Fabrication of miniature elastomer lenses with programmable liquid mold for

smartphone microscopy: polydimethylsiloxane within situ curvature control, Journal of Biomedical Optics 23 (2) 2018

[10] Lee S., Lee Ch., Verkade R.,V., Cheon G.W., Kang J.,V., Common-path all-fiber optical coherence tomography probe based on high-index elliptical epoxy-lensed fiber, Optical Engineering 58 (2) 2019

[11] Tkaczenko W.M., Kotiuk C.O., Czerewań E.O. (Analysis of the simplest optical center systems) in Ukrainian, Collection of scientific works of the Faculty of Physics and Mathematics, Drokhobych State Pedagogical University of Ukraine 2023, no. 13, pp. 34-38

[12] Puchalski J., Warsza Z.L.: The method of fitting a non-linear function to data of measured points and its uncertainty band. *-in Polish*. PAR (*Pomiary Automatyka Robotyka*) n. 3_2023 pp. 45 –55, doi:10.14313/PAR_249/45

[13] Warsza Z.L., Puchalski J.: *Assessment of the uncertainty of selected nonlinear functions determined from measurements by the linear regression method*. World Seminar AMCTM XIII Sept. 2023 Sarajevo. Monograph: Pavese F., Bošnjaković A., Eichstädt S., Forbes A. B., Alves e Sousa J. (eds.): Advanced Mathematical and Computational Tools in Metrology and Testing XIII. Series on Advances in Mathematics for Applied Sciences, Vol. 94 © 2025 World Scientific Publ. Co. p. 288–307, doi.org/10.1142/9789819800674_0027

[14] Warsza Z.L., ‚Puchalski J., Więcek T., Method for fitting a nonlinear function to measurement data and its uncertainty band.- in Polish (*Metoda dopasowania funkcji nieliniowej do danych pomiarowych i jej pasmo niepewnośc*i) in: Monograph of XV Scientific Conference SP'2024 Measurement Systems in Research and Industry, Łagów 9-12.06.2024 Zielona Góra University Pl. pp.151-154

[15] Warsza Z.L., Puchalski J., Więcek T., Examples of fitting nonlinear functions by linearization with changes variables. – in Polish (*Przykłady dopasowywania charakterystyk nieliniowych metodą zmiany zmiennych i ich niepewności*). in Monograph of 56 MKM Conference: Rząsa M.R (ed.) Selected Aspects of Metrology. Opole University of Technology, 2024 pp. 29-44, ISN 1429-6063, ISBN978-83-66903-71-5

[16] Warsza Z.L., Puchalski J., Więcek T., *Metoda ze zmianą zmiennych dla dopasowania funkcji nieliniowych do danych pomiarowych i ich niepewności*. Przegląd Elektrotechniczny 2024 nr12 s.252-259 in Polish, doi:10.15199/48.2024.12

[17] Warsza Z.L., Puchalski J., Więcek T., Novel Method of Fitting a Nonlinear Function to Data of Measurement Based on Linearization by Change Variables, Examples and Uncertainty, Metrology 2024, 4, pp. 718–735. doi.org/10.3390/metrology4040042

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