HEURISTIC METHOD OF ASSEMBLY SCHEDULING OF MULTI-OPTION PRODUCTS FOR ASSEMBLY LINE WITH INTERMEDIATE BUFFERS

Heurystyczna metoda harmonogramowania montażu wielowariantowych produktów linii montażowej z buforami międzyoperacyjnymi

Евристический метод чередования сборки многовариантных изделий в сборочной линии с межоперационными магазинами

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A b s t r a c t: The method intended to build the possibly shortest assembly schedules is presented in the paper. The method is constructed for assembly lines without parallel machines. The intermediate buffers are located between the assembly machines. A flow of products of different types is unidirectional in the assembly line. The proposed method is a heuristic because the fixed tasks are characterized by a large size and considerable computational complexity. It is a relaxation heuristic. The linear mathematical model is used in the mentioned heuristic method. The method is constructed for assembly scheduling for multi-option products. Assembly of various types of products in different variants is the reply to the contemporary challenges faced by the manufacturers, who try to satisfy the demands of individual customers. The planned downtimes of machines are regarded in the method, for example downtime of machines connected with maintenance. The results of computational experiments with proposed heuristic method are presented. The described heuristic method is compared with optimal method based on the integer programming. The fixed makespans and computational fixed times using heuristic method and optimal method are compared.
K e y w o r d s : heuristic, relaxation, assembly line, optimization, scheduling, linear programming

S t r e s z c z e n i e: Przedstawiono metodę przeznaczoną do budowy najkrótszych harmonogramów montażu produktów w liniach montażowych bez maszyn równoległych. Konfiguracja linii montażowej uwzględnia obecność buforów międzyoperacyjnych pomiędzy maszynami. Przepływ równocześnie montowanych produktów różnych typów jest jednokierunkowy. Ze względu na rozmiary rozwiązywanych problemów i związaną z tym złożoność obliczeniową metoda jest heurystyką. Jest to heu¬rystyka relaksacyjna, w której wykorzystano model zadania programowania liniowego.Metoda dotyczy produktów wielowariantowych. Produkty danego typu mogą różnić się warian¬tami wykonania – specyficznymi cechami, uwzględniającymi wymagania odbiorców. Metodę wyróżnia także uwzględnienie planowanych przestojów maszyn, np. przeznaczonych na konserwację. Zaprezentowano wyniki eksperymentów obliczeniowych, za pomocą których oceniono jakość opracowanej metody. Długości harmonogramów wyznaczanych za pomocą przedstawionej metody porównano z długościami harmonogramów optymalnych, znanymi dzięki zastosowaniu modelu zadania programowania całkowitoliczbowego. Porównano także czasochłonność obliczeń.
S ł o w a k l u c z o w e : heurystyka, relaksacja, linia montażowa, optymalizacja, szeregowanie operacji, programowanie liniowe

Introduction – the reasons for construction of heuristic

The method for construction of assembly scheduling, as suggested in the present paper, may be assigned to the approximate methods; it is the heuristics. The term heuristic originates in Greek language, it comes from the word *heurisko*, that means *I find*, *I discover*. The heuristics are employed first of all in the cases where the complete algorithms are too expensive due to technical reasons [2]. The mentioned expensiveness – in the case of the submitted method – concerns the dimensions of the problems to be solves and the computational complexity.

In construction of the possibly shortest assembly scheduling, the multi-option products were considered. The products of a given type that are assembled in various variants differ in respect of their distinguishing features, e.g. additional componential parts (e.g. handlers), shape and dimensions of case, or other factors affecting their external appearance and utility properties. The consideration of individual requirements of customers, being the undoubted advantage of the developed method, results in relatively great extent of the problems to be solved. The extent of the problem is understood as integer number, being a measure of the quantity of input data which characterize a given problem [6]. The input data include, *inter alia*, parameters concerning various variants of the products and, also, parameters of assembly line. The extent of the problem has an influence on the quantity of computer resources, being necessary for solving the problem, that is, computational complexity. The mentioned resources include memory (memory complexity) and time (time complexity).

In connection with the above, in case of the problems of a great extent, the search for the optimum solution is often unprofitable. Due to this reason, the heuristic method was constructed owing to which it is possible

– in a relatively short time – to find the solutions which may be charged with a certain deviation from optimum. It is significant in the case of the necessity to repeat the scheduling – to determine a new scheduling in a short time.

The examples of the application of heuristics in the assembly scheduling may be found in the research papers [1], [3] and [6]. The method suggested in the present study is distinguished by the consideration of multi-option products and, also, the planned downtimes.

The heuristic method was described in chapters 2 and 3. The successive chapter is dedicated to the evaluation of the discussed heuristic – the comparison of the set optimum solutions and the heuristic ones.

Formulation of the problem and the concept of its solution

Unidirectional assembly line without parallel machines is given. Between the machines, there are intermediate buffers with limited capacities where the products may wait for performance of the successive operations. In the assembly line system, we may have simultaneously assembled multi-option products of different types. Some machines may be omitted by the product. The exemplified configuration of assembly line which the developed method is referred to is illustrated in Fig.1.

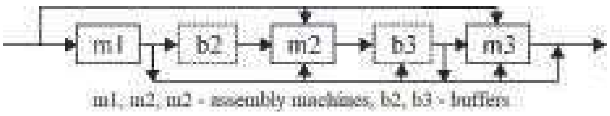

Fig. 1. Unidirectional assembly line with intermediate buffers
Rys.1. Jednokierunkowa linia montażowa z buforami międzyoperacyjnymi

In case of the above assembly line, the shortest scheduling of performing the assembly operations should be constructed.

The method concerning the solution of the described problem is based on the monolithic approach. The assignment of operations to machines and distribution of operation in time is performed simultaneously. The block diagram of the discussed method is given in Fig.2.

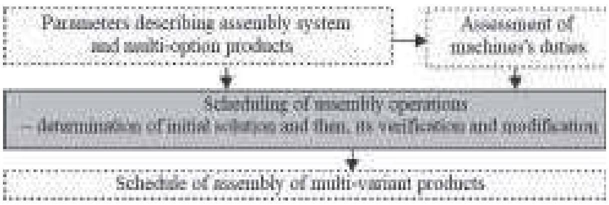

Fig. 2. Block diagram of the heuristic method
Rys.2. Schematy blokowy heurystycznej metody

Table 1. Summary of sets, parameters and variables
Tabela 1. Zestawienie oznaczeń zbiorów, parametrów i zmiennych

Sets:	
I	– Set of assembly machines: $I = \{1, ..., M\}$;
J	– Set of types of assembly operations: $J = \{1, ..., N\}$;
K	– Set of types of assembled products: $K = \{1, , W\}$;
L	– Set of time intervals: $L = \{1, ..., H\}$;
S	– Set of indices of multi-variant products;
Ij	– Set of machines capable of performing the operation j Î J;
J^1	– Set of basic operations, J^1Ì J;
J^2	– Set of additional operations (rendering the specific features to the multi-option products), J^2 Ì J;
J^C	– Set of assembly operations consisting in additional assembly of a part, taken from feeder, J^C Ì J;
O^1	– Set of pairs (k, j), in case of which the basic operation j Î $J1$ concerns the product of k Î K type;
O^2	– Set of pairs (s, j), in case of which additional operation j Î $J2$ concerns s Î S product;
R^1	– Set of ordered triples (k, r, j), in which performed successively basic operations r, j Î J^1 concern product of k type;
R^2	– Set of ordered triples (s, r, j), in which successively performed operations r, j Î J concern product s and at least one operation belongs to set J^2;
T	– Set of ordered pairs (s, k), where product s is of k type;
Parameters:	
a_{ij}	– Working space of machine i utilised for performing operation of j type;
b_i	– Working space of machine i, where feeders of componential parts may be placed;
d_i	– Capacity of intermediate buffer placed before machine i;
$g_{\tau i}$	– Time of product's transport between machines τ and i;
p^1_{jk}	– Time of performing basic operation j Î J^1 concerning product of k type;
p^2_{js}	– Time of performing additional operation j Î J^2 concerning s product;
n_{il}	= 1, if machine i is available in time interval l, otherwise nil = 0;
Variable, the values of which are determined in the particular iterations e:	
x^e_{ij}	= 1, if type of operation j is assigned to machine i, otherwise x^e_{ij}
q^e_{ijsl}	= 1, of in time interval l operation j concerning product s is performed in machine i; otherwise q^e_{ijsl};
y^e_{isl}	= 1,if in time interval l product s is found in intermediate buffer placed before machine i, otherwise y^e_{isl} = 0;
w^e_{is}	– Time of commencing performance of operations concerning product s in machine i;
z^e_{is}	– Time of completing the performance of operations concerning product s in machine i;

Fig.2 shows that the parameters, describing machine ark and multi-option products, are utilized in the procedure, intended for assessment of machine duties. It results from the consideration of the planned downtimes of machines. In the heuristic destined for solving the problem, the linear mathematical model, constructed by the author of the present paper, concerning the scheduling of assembly line with intermediate buffers was employed. In the discussed model, the conditions of integer numbers of decisive variables were omitted. It is a characteristic of relaxation heuristics. In the developed method, the rules of rounding of variables, informing on assignments of particular assembly operations to the machines in given time intervals, were specified. The discussed method includes also procedures of verification and modification o the schedule to be constructed, e.g. those ones concerning behaviour of sequence limitations (performance of operations in accordance with each sequence, in unidirectional assembly line), lack of collision in assembly (in a given moment on a specified machine there is no more than one operation performed; in the case of the product, also, in a given moment no more than only one assembly operation assigned to this product is performed).

The detailed description if the method is given in the next chapter. Omitting of the conditions of integer numbers of variables in mathematical model affected very positively the time consumption of the calculations what was indicated in chapter 3 concerning the computational experiments, carried out on the developed method.

Mathematical description of relaxation heuristic

In the relaxation heuristic, the sets and parameters compiled in Tab.1 were considered.

The parameters described in Tab.1 are the input data considered in the heuristic algorithm. The values of variables, being defined in the mentioned table, determined in the last iteration are the solution. Here we have the relaxation heuristic:

Step 1. Linear relaxation of mathematical model, initial solution

Apply the procedure for determination of the number of time intervals H, as published in the paper [5]. Adopt the number of iteration e = 1 and solve the task formulated in linear mathematical model which the function of aim concerns (1) and constraint (2) – (20).

Minimize:

$$\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{s\in S} l q^{e}_{ijsl} \tag{1}$$

Subject to:

$$\sum_{i\in I}\sum_{j\in J} q^{e}_{ijsl} \le 1;\ l\in L; s\in S \tag{2}$$

$$\sum_{j\in J}\sum_{s\in S} q^{e}_{ijsl} \le n_{il};\quad i\in I;\ l\in L \tag{3}$$

$$\sum_{i\in I_j}\sum_{l\in L: n_{il}=1} q^{e}_{ijsl} = p^{1}_{jk};\ j\in J^{1}; (s,k)\in T: (k,j)\in O^{1} \tag{4}$$

$$\sum_{i\in I_j}\sum_{l\in L: n_{il}=1} q^{e}_{ijsl} = p^{2}_{js};\ j\in J^{2}; s\in S: (s,j)\in O^{2} \tag{5}$$

$$q^{e}_{ijsl} + q^{e}_{\tau jsf} \le 1;\quad \tau, i\in I;\ j\in J;\ l,f\in L;\ s\in S;\ \tau\ne i \tag{6}$$

$$l q^{e}_{ijsl} - f q^{e}_{\tau r s f} \le \sum_{\eta\in L: f<\eta<l}\left(q^{e}_{ijs\eta} + q^{e}_{\tau rs\eta}\right) + 1 + (H+1)\left(1-q^{e}_{ijsl}\right)$$

$$i,\tau\in I;\ (s,k)\in T;\ l,f\in L;\ (k,r,j)\in R^{1} \vee (s,r,j)\in R^{2} \tag{7}$$

$$x^{e}_{ij} \ge q^{e}_{ijsl};\ i\in I;\ j\in J;\ l\in L;\ s\in S \tag{8}$$

$$x^{e}_{ij} = 0;\quad i\notin I_j;\ j\in J \tag{9}$$

$$a_{ij} x^{e}_{ij} \le b_i;\quad i\in I;\ j\in J' \tag{10}$$

$$i q^{e}_{ijsl} \ge \tau q^{e}_{\tau r s f} - (H+1)\left(1-q^{e}_{ijsl}\right);\ i,\tau\in I; (s,k)\in T; l,f\in L; (k,r,j)\in R^{1} \vee (s,r,j)\in R^{2} \tag{11}$$

$$l q^{e}_{ijsl} - f q^{e}_{\tau r s f} \ge 1 + g_{ri} - (H+1)\left(1-q^{e}_{ijsl}\right)$$

$$i,\tau\in I;\ f,l\in L;\ \left((s,k)\in T; (\kappa,r,j)\in R^{1}; \kappa=k\right) \vee (s,r,j)\in R^{2} \tag{12}$$

$$w^{e}_{is} \ge l q^{e}_{ijsl} - \sum_{f\in L}\sum_{r\in J} q^{e}_{irsf} + 1 - (H+1)\left(1-q^{e}_{ijsl}\right);\quad i\in I;\ j\in J;\ l\in L;\ s\in S \tag{13}$$

$$w^e_{is} \le l q^e_{ijsl} + (H+1)(1-q^e_{ijsl}); \quad i \in I;\ j \in J;\ l \in L;\ s \in S \tag{14}$$

$$z^e_{is} \ge l q^e_{ijsl} - (H+1)(1-q^e_{ijsl}); \quad i \in I;\ j \in J;\ l \in L;\ s \in S \tag{15}$$

$$l q^e_{ijsl} - f q^e_{\tau rsf} - 1 \le g_{\tau i} \sum_{\delta \in L;\ f<\delta<l} \left(q^e_{ijs\delta} + q^e_{\tau rs\delta}\right) + \sum_{\delta \in L} y^e_{is\delta} + (1+H)(1-q^e_{\tau rsf})$$

$$\tau, i \in I;\ \tau < i;\ (s,k) \in T;\ f, l \in L;\ f < l;\ (k,r,j) \in R^1 \vee (s,r,j) \in R^2 \tag{16}$$

$$l y^e_{isl} \ge z^e_{\tau s} + 1 + g_{\tau i} - (H+1)(1-y^e_{isl}); \quad \tau, i \in I;\ l \in L;\ s \in S;\ \tau < i \tag{17}$$

$$l y^e_{isl} \le w^e_{is} - 1 + (H+1)(1-y^e_{isl}); \quad i \in I;\ l \in L;\ s \in S \tag{18}$$

$$\sum_{s \in S} y^e_{isl} \le d_i; \quad l \in L;\ v \in V;\ i > 1 \tag{19}$$

The presented model is intended for determination of the shortest schedules owing to minimization of sum (1). The constraints concerning the discussed model ensure: (2) – performance in a given moment of no more than one assembly operation concerning a defined product; (3) – performance of no more than one operation by assembly machine in a given moment if the machine is available in this moment; (4) and (5) – distribution of basic (4) and additional (5) operations into machines; (7) – indivisibility during the operations concerning a given product, performed in a specified machine; (8) – determination of the assignment of types of operations to the machines; (9) – elimination of the assignment of operations to the wrong machines; (10) – preservation of the limited working space of each of machines: (11) – unidirectional flow of products throughout the assembly line; (12) – consideration of sequence limitations in performing the operations and ensuring the time for transportation of the products between the machines: (13) and (14) – determination of the times of commencing the performance of operations concerning the specified products in the particular machines and (15) – determination of the times of completing the performance of these operations; (16) – determination of time intervals in which the assembled product must remain in buffer during the time period, preceding the next operation – owing to relation (17) and after transportation to this machine – owing to condition (18) with consideration of limited capacity of buffers what ensures dependence (19).

In the described model, the integer number conditions were omitted. To determine the initial solution, the auxiliary variables have been introduced:

v_{ijs}=1, if operation j concerning product s is performed in machine j, otherwise v_{ijs} = 0;

t^e_{sij}, c^e_{sij} - times of: commencing, completing of operation j for product s, performed in machine i

To determine values of the mentioned variables, assign v_{ijs} to the mentioned machine i capable of performing operation j concerning product s, in the case of which value $\sum_{l \in L} q^e_{ijls}$ is the highest one. If value of this sum is the same in case of few machines, choose the machine capable of performing operation j with lower index i.

Assume number of iteration e: = 2. Make acc. to (20) rounding of the variables, determine the times of commencing the operations (21) and times of completing the operations (22).

$$q^e_{ijls} = \mathrm{round}\left(q^{e-1}_{ijls}\right); \quad i \in I;\ j \in J;\ l \in L;\ s \in S \tag{20}$$

$$t^e_{ijs} = v_{ijs} \cdot \min_{l \in L, \tau \in I}\left(l q^e_{ijls}\right); \quad i \in I;\ j \in J;\ s \in S \tag{21}$$

$$c^e_{ijs} = \begin{cases} t^e_{ijs} + p^1_{jk} - 1 & i \in I;\ j \in J^1;\ (s,k) \in Z:\ (k,j) \in O^1 \\ t^e_{ijs} + p^2_{js} - 1 & i \in I;\ j \in J^2;\ s \in S:\ (s,j) \in O^2 \end{cases} \tag{22}$$

Assume e: = e + 1 and remember loads of particular machines in the particular l periods, using 23 and go to step 2.

$$q^e_{ijls} = \begin{cases} 1, \text{ gdy } t^{e-1}_{ijs} \geq l \leq c^{e-1}_{ijs} \\ 0, \text{ inaczej} \end{cases}; \; i \in I; \; j \in J; \; l \in L; \; s \in S \tag{23}$$

Step 2. Verification of direct succession of operations performed in a given machine concerning the same product

a) Check whether in the case of each pair of successively performed operations r, j, concerning s product of k type where $(k, j, r) \in R^1$ or $(s, r, j) \in R^2$ performed in the same machine, the equation (24) is satisfied. If so, go to step 5; if not – go to step 5b.

$$t^e_{jrs} - c^e_{ijs} = 1; \quad i \in I; \; (s,k) \in T; \; r, j \in J; \; v_{ijs} = v_{irs} = 1 : (k,r,j) \in R^1 \vee (s,r,j) \in R^2 \tag{24}$$

Let e: =e + 1. Remember each product s' (mark it in such a way) that does not satisfy condition (24) in case of machine i' (mark it in such a way) and then, determine the set of operations performed in this machine, mark it as J'. Mark with j' the first operation, performed in machine i' concerning s' product. The times of commencing and completing of these operations, in the case of which the equation (24) is satisfied remain unchanged and are remembered owing to (25) and (26). In the case of i operations of the products which have not met the constraint (24), there are determined acc. to (27) and (28) such times of beginning and ending the operations as to satisfy constraint (24)

$$t^e_{ijs} = t^{e-1}_{ijs} \text{ dla } \left(i \in I \setminus \{i'\}; \, j \in J\right) \vee \left(i = i'; \, j \in J \setminus J' \vee j = j'\right); \; s \in S \tag{25}$$

$$c^e_{ijs} = \begin{cases} t^e_{ijs} + p^1_{jk} - 1 \text{ dla } \left(i \in I \setminus \{i'\}; \, j \in J^1\right) \vee \left(i = i'; \, j \in J^1 \setminus J' \vee j = j'\right); \; (s,k) \in T \\ t^e_{ijs} + p^2_{js} - 1 \text{ dla } \left(i \in I \setminus \{i'\}; \, j \in J^2\right) \vee \left(i = i'; \, j \in J^2 \setminus J' \vee j = j'\right); \; s \in S \end{cases} \tag{26}$$

$$t^e_{ijs} = c^e_{irs} + 1 \text{ dla } i = i'; \, r \in J'; \, j \in J' \setminus \{j'\}; \, s = s'; \left(\left(k,r,j\right) \in R^1 \wedge (s,k) \in T\right) \vee (s,r,j) \in R^2 \tag{27}$$

$$c^e_{ijs} = \begin{cases} t^e_{ijs} + p^1_{jk} - 1 \text{ dla } i = i'; \; j \in J' \setminus \{j'\}; \; (s,k) \in Z; \; s = s' \\ t^e_{ijs} + p^2_{jk} - 1 \text{ dla } i = i; \; j \in J' \setminus \{j'\}; \; s \in S; \; s = s' \end{cases} \tag{28}$$

Update acc. to (29) information on loading of machines. Then, undo denotation of machines, operations, products and set of operations (i', j', s', J') and go to step 3.

$$q^e_{ijls} = \begin{cases} 1, \text{ gdy } t^e_{ijs} \geq l \leq c^e_{ijs} \\ 0, \text{ inaczej} \end{cases}; \; i \in I; \; j \in J; \; l \in L; \; s \in S \tag{29}$$

Step 3. Verification of collision-free condition of performing the operations

a) Let machine index i : = 0. Go to step 3b.
b) Let i : = i + 1 and index of time interval l : 0. Go to step 3 c.
c) Let l : l + 1. If the condition (30) is met, i.e. in a given period l, machine i performs no more than one operation, go to step 4; if not, go to step 3 d.

$$\sum_{j \in J} \sum_{s \in S} q^e_{ijls} \leq n_{il} \tag{30}$$

d) Choose only one product s which meets the equation $q^e_{ijls} = 1 \; (j \in J)$ and determine s' while making the choice of this product according to lexicographic order: 1) the product which met the equation $q^e_{ijls} = 1 \; (j \in J)$ in the previous iteration and was not marked with s'; 2) the product with the lowest value $t^e_{ijs} \; (j \in J)$; 3) the product with the highest value $\sum_{j \in j} \sum_{l \in L} q^e_{ijls}$; 4) the product with the lowest index s.

Determine j' operation concerning s' product, performed in the l period in i machine. Assume e: = e + 1. When applying (31), determine the number of time intervals b, in which j' operation requires load of machine i in the period from l until completion of the discussed operation.

$$b = c_{ijs'}^{e-1} - l + 1 \tag{31}$$

On the base of (32), determine element of D set – ordered pairs (s, j) where j operation concerns s product. Set D includes products and their relating operations in the case of which it is necessary to make modification of scheduling – ascribe the later time of performing the operations for these products (prolongation of the scheduling by b periods).

To ensure the collision free condition of assembling, modify the scheduling acc. to (33). Update the times of commencing and completing the performance of operations in accordance with the equations (21) and (22). Undo the denotation of operations and products (j', s') and go to step 4.

$$D = \left\{(s,j) : s \in S;\ j \in J \setminus \{j'\} : q_{ijfs}^{e-1} = 1;\ f \in \langle l, l+b-1 \rangle \right\} \tag{32}$$

$$q_{\tau j\rho s}^{e} = \begin{cases} q_{\tau j\rho s}^{e-1} \text{ dla } \left((\tau \in I \setminus \{i\}; (s,j) \in D) \vee (\tau = i; j \in J; (s,j) \notin D)\right) \\ q_{\tau jfs}^{e-1} \text{ dla } \tau = i; (s,j) \in D; \rho \geq l+b,\ f = \rho - b \end{cases} ; \rho \in L; s \in S \tag{33}$$

Step 4. Verification of behaviour of limitations concerning the sequence of performing the operations

a) If the verified constraint (34) concerning the sequence limitations is satisfied for j operation performed in i machine and its predecessor – operation r, performed in τ machine, go to step 5, otherwise, go to step 4b.

$$t_{ijs}^{e} - c_{\tau rs}^{e} - 1 \geq g_{\tau i};\ \tau, i \in I;\ s \in S;\ t_{ijs}^{e} \leq l;\ \left((k,r,j) \in R^1 \wedge (s,k) \in Z\right) \vee (s,r,j) \in R^2 \tag{34}$$

b) Mark operation which does not satisfy constraint (34) with letter j', and mark the product relating to the discussed operation with letter s'. Determine b – minimum number of periods by which value $t_{ij's'}^{e}$, should be increased as to preserve limitation (34). Assume e : =e + 1 and modify scheduling acc. to (35). Update the times of commencing and completing the operations concerning products acc. to (21) and (22). Undo the denotation of operations and products (j', s') and go to step 5.

$$q_{\tau j\rho s}^{e} = \begin{cases} q_{\tau j\rho s}^{e-1} \text{ dla } \left((\tau \in I \setminus \{i\}; j \in J) \vee (\tau = i; j \in J \setminus \{j'\})\right); \rho \in L; s \in S \\ q_{\tau jfs}^{e-1} \text{ dla } \tau = i; j = j' : \rho \in L;\ \rho \geq l+b;\ f = \rho - b;\ s = s' \end{cases} \tag{35}$$

Step 5. The condition of halting for the so-far existing stages of verification and improvement of scheduling

If $l < \max_{\tau \in I, j \in J, s \in S} c_{\tau js}^{e}$, go to step 3c, otherwise check: i <M (M – number of machines). If the relation is satisfied, go to step 3b; otherwise go to step 6˙

Step 6. Verification of utilizing intermediate buffers and availability of machines

Assume e: = e + 1 and solve the system of inequalities (13) – (19) in order to determine the loads of intermediate buffers. When analyzing the successive time intervals and loads of machines and loads of intermediate buffers as well as a limited availability of machines (checking the constraint (3)), verify and modify a scheduling in such a way as to have the admissible number of products in the particular buffers in a given moment and the machines loaded in the periods of their availability. In the successive iterations, analogically as in the case of equation (31) in step 3, determine the number of time intervals b, by which the operations which do not satisfy the described constraints, should be "shifted". If a few products do not meet the mentioned constraints (in a given iteration), employ lexicographic order, as being described in step 3 in order to choose one product. Modify the scheduling analogically to constraint (33) and update the times of performing operations, applying (21) and (22).

The times of commencing and completing the performance of the particular operations, as determined in the last iteration – on the grounds of equations (21) and (22) are the solution.

The computational experiments – relaxation heuristic and the accurate method

The submitted heuristic algorithm (algorithm H) has been subjected to verification. The determined schedules, using the mentioned algorithm were compared to the scheduling constructed with the use of the precise method (method M). In the accurate method, there was solved a task, formulated in mathematical model (1) – (19), supplemented by the constraints, ensuring the binarity of decision variables – thus, the integer programming was employed.

The computational experiments included 4 groups of test tasks. In the case of each of the groups, 25 test tasks were solved. In the computation, GUROBI [6] was used. The parameters of the groups of test tasks and the results of the experiments were presented in Tab.2. In the mentioned table, there were presented the mean values of 2 indicators, defined in the equations (36), indispensable for evaluation of heuristic: f_1 – destined for comparison of the length of scheduling, and f_2 – for comparison of the computation times using heuristic method H and accurate method M. in the mentioned equations C_{max} is a length of the scheduling and CPU represents the time of computations.

$$f_1 = \frac{C^H_{max} - C^M_{max}}{C^M_{max}} \cdot 100\%, \quad f_2 = \frac{CPU^H}{CPU^M} \cdot 100\% \qquad (36)$$

Tab.2. Parameters of groups of tasks and the results of computational experiment

Group of tasks	Parameters of group of tasks					The results of experiments	
	M	N	W	S	H	f_1 [%]	f_2 [%]
1	3	10	3	6	16	3,5	0,37
2	4	12	4	8	18	3,7	0,33
3	4	14	5	10	20	3,6	0,27
4	5	16	6	12	22	4,3	0,26

Number of : M – machines, N –types of assembly operations, W – types of products, S – products, H – time intervals

The mean values of indicators f_2 show multiple abbreviation of the time of computations in the case of applying hierarchic method for solution of test tasks – the schedulings were determined by 270 – 380 times quicker than in the case of the application of accurate method. The discussed abbreviation of the time of computations had however certain deviation from optimum, amounting to 3.5 – 4.3 % what is shown by the values of f_1 indicators.

Final remarks

The unquestionable advantage of the presented method consists in consideration of multi-option products, in the assembly schedulings, constructed in a relatively short period of time. A quick solution of the formulated problem has been reached owing to the application of relaxation technique. This relevant favourable feature of the presented method has, however, certain deviation from optimum what is characteristic of the approximate methods. The conducted computational experiments allowed measuring of defects and advantages of the suggested method.

Owing to the described above features, the presented method is recommended mainly for the operational planning in the case of assembly of the products which consider the individual requirements of customers. It is also suitable in the case of re-scheduling – when there is a necessity to construct a new scheduling in relatively short time.

Mathematical model, as presented in this paper, and supplemented by the conditions of integers of decision variable, may be utilized in construction of optimum schedulings, especially in the case of solving the small dimension problems.

The discussed model may be modified, adopted to the varying conditions of the assembly process and employed in innovative operations in automated assembly [4].

REFERENCES

[1] Bhongade A.S., Khodke P.M.: Heuristics for production scheduling problem with machining and assembly operations. International Journal of Industrial Engineering Computations, 3 (2), 2012, 185–198.

[2] Campo C., Pauser S., Steiner S., Vetschera R.: Decision making styles and the use of heurustics in decision making. Journal of Business Economics, 86 (4), 2016, 389–412.

[3] Jeong B., Sim S., Jung H.: Heuristics for assembly operation scheduling problem in flexible assembly systems. International Journal of Advanced Manufacturing Technology, 29 (9–10), 2006, 1033–1040.

[4] Łunarski J.: Doskonalenie procesów projektowania działaniami innowacyjnymi w zautomatyzowanej produkcji masowej. Technologia i Automatyzacja Montażu. Zeszyt 3-2017. Warszawa 2017, 4–5.

[5] Magiera M.: Monolityczna metoda planowania montażu dotyczącego wielowariantowego sprzętu elektrycznego i elektronicznego. Przegląd Elektrotechniczny, 2017 R. 93, nr 8, 192–195.

[6] Magiera M.: Wybrane metody planowania przepływów produktów przez linie produkcyjne i łańcuchy dostaw. Wydawnictwa AGH, Kraków 2016.

[7] www.gurobi.com (Gurobi Optimizer, 2018).

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