## ANALYSIS OF VON MISES HYPOTHESIS USEFULNESS FOR ADHESIVE LAYER EFFORT ESTIMATE

### Analiza przydatności hipotezy Hubera do oceny wytężenia spoin klejowych

#### Jan GODZIMIRSKI, Sławomir TKACZUK

A b s t r a c t: In the adhesive layer, uneven stress distribution and their complex state usually occur. Development of methodology for the analysis of adhesive layer was an aim of the experimental studies and numerical calculations. Specimens with adhesive layers which were similar to each other in terms of even distribution of stresses, were used in trials. This condition was met by using cylindrical oblique specimens consisting of two elements, obtained by cutting symmetrically at an angle of 45° a cylinder with a diameter of 12 mm and a height of 20 mm and bonding them adhesively on the cutting surfaces. The result of the experiment showed a greater than 2.26 to 2.92 strength of compressed specimens compared to stretched ones. It allows to conclude that the Mises hypothesis cannot be directly used for analysis of the state of stresses in adhesive joints. Numerical calculations were performed. For those, the model of the specimens was put under tension and compression with mean values of forces determined experimentally for specimens bonded with Epidian 57 adhesive. Conclusions were allowed to be formulate by conducted trials and calculations. Structural adhesives exhibit higher compressive strength rather than tensile strength. Consequently they exhibit indirect properties between metallic elastic plastics and brittle ceramic bodies. The criterion for destroying an adhesive bond loaded temporarily may be that the von Mises stress exceeds the absolute value of the tensile strength of the adhesive or compression. However it should be taken into account that those strengths have different values.

Keywords: adhesive joints, effort of adhesive layers, effort hypothesis

Streszczenie: W spoinach połączeń klejowych występuje zazwyczaj nierównomierny rozkład naprężeń i ich złożony stan. Celem prowadzonych badań eksperymentalnych i obliczeń numerycznych było opracowanie metodyki analizy wytężenia spoin klejowych. W badaniach zastosowano takie próbki, w spoinach których występuje zbliżony do równomiernego rozkład naprężeń. Warunek ten spełniają próbki cylindryczne skośne składające się z dwóch elementów uzyskanych przez przecięcie walca o średnicy 12 mm i wysokości 20 mm, symetrycznie pod kątem 450 i sklejenie ich na powierzchniach cięcia. Przeprowadzony eksperyment wykazał większą od 2,26 do 2,92 wytrzymałość próbek ściskanych w porównaniu do rozciąganych, co pozwala stwierdzić, że hipoteza Misesa nie może być stosowana bezpośrednio do analizy stanu wytężenia spoin klejowych. Przeprowadzono obliczenia numeryczne, w których model próbki obciążono na rozciąganie i ściskanie średnimi wartościami sił wyznaczonych eksperymentalnie dla próbek łączonych klejem Epidian 57. Przeprowadzone badania i obliczenia pozwoliły sformułować wnioski. Kleje konstrukcyjne cechuje większa wytrzymałość na ściskanie niż na rozciąganie, a więc wykazują one właściwości pośrednie pomiędzy metalicznymi ciałami sprężysto-plastycznymi i ceramicznymi kruchymi. Kryterium zniszczenia spoiny klejowej obciążonej doraźnie może być przekroczenie przez naprężenia von Misesa bezwzględnej wartości wytrzymałości kleju na rozciąganie lub na ściskanie, ale należy uwzględniać, że te wytrzymałości mają różne wartości

Słowa kluczowe: połączenia klejowe, wytężenie spoin klejowych, hipotezy wytężenia

#### Introduction

Adhesive bonding is a typical method of joining applied in numerous branches of industry [1, 3, 4]. It is a particularly attractive technique of assemblance for applications, in which weight gain is at a premium, such as land and air transport [9]. In adhesive layers there are complex states of stress. Its distribution is usually uneven. Analytical solutions for single lap joints have been developed since 1938, when Volkersen analysed the shear-lag problem [13]. He ignored the bending moments applied to the joint because of the eccentricity of the load. The first solutions for peel stresses as well as for shear stresses were created by Goland and Reissner [2]. Afterwards their theory was improvedde[5, 6, 16]. A great normal stress concentration in adhesive layers of joints subjected to peeling was pointed out by Jouwersma [10], Lamb [12], Yurenka [15] and Gardon [7]. The stress concentration is

a cause of low strengths of such joints. However, only the finite element (FE) method enabled to determine complex states of stress in adhesive layers. An attempt to apply the FE method for stress analysis in the adhesive layer was already undertaken in the year 1971 [14]. Nowadays the FE method is often used for examining the strength of the adhesive joints [8, 11].

Therefore calculations of adhesive joints strength demand to determine adhesive layer effort according to certain theory and a comparison of reduced stresses with the adhesive strength. Conducted researches proved that extensively used von Misses hypothesis is useful for metallic materials. Moreover, the von Misses hypothesis is also commonly and unquestioningly used for an analyses of adhesive bonded joints. Numerous constructional adhesives show indirect properties between an elastic-plastic metal and a brittle body. Therefore, the adhesives

show higher compression strength in comparison with tensile strength. The von Misses hypothesis does not reassert for a brittle body. Thus the von Misses theory should be used deliberately for a strength analyses of adhesive bonds.

#### **Experimental research**

In the trials, it was decided to use such specimens, in which there is a similar to even distribution of stresses. The numerical analysis demostrated that this condition is met by cylindrical oblique specimens consisting of two elements obtained by cutting symmetrically at an angle of 45 ° a cylinder with a diameter of 12 mm and a height of 20 mm, (Fig. 1) and adhesively bonding them on the cutting surfaces (Fig. 2).

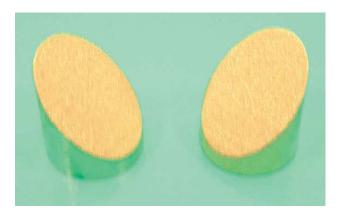


Fig. 1. Elements of cylindrical skew specimens Rys. 1. Elementy próbek cylindrycznych skośnych



Fig. 2. Adhesively bonded cylindrical skew specimen for compressive test and specimens after tensile test Rys. 2. Sklejona próbka cylindryczna skośna przeznaczona do badań na ściskanie oraz zerwana, obciążona na rozciąganie

The prepared specimens were loaded by compression and stretching. In order to stretch the higher, specimens were adhesively bonded into cylindrical brackets enabling axial-symmetrical fixing them into the holders of the testing machine.

Parts of the specimens were made of aluminum alloy AW 2017-T3 by spark erosion cutting the roller elements. The adhesively bonded surfaces of the

specimens were sandblasted, washed with gasoline and dried before the adhesive was applied. During the curing of the adhesive layers, the specimen's elements were placed in the matching holes of Teflon molds and pressed together under load 10 N.

If the von Misses hypothesis is appropriate for analyzing the stresses of adhesively bonded joints, then the strength of tensile and compressed samples should be similar. This requirement arise from the fact that if tensile and compressed specimens have the same value of force, there will be normal and tangential stresses in the adhesive layers with the same absolute value but with differed sign. After stretching the three components of normal stresses are positive and negative components under compression.

From the formula describing the reduced stresses by the Mises hypothesis:

$$\sigma_{red} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \sigma_y - \sigma_y \sigma_z - \sigma_z \sigma_x + 3(\tau_y^2 + \tau_y^2 + \tau_y^2)}$$
(1)

it follows that the value of reduced stresses should be the same for positive and negative normal stresses if these stresses have the same absolute value.

Experimental tests were conducted for four adhesive and three epoxy: Epidian 57 / Z1, Poxipol and Raychem and a cyanoacrylate Dragon. Three pressed specimens and three stretched ones were adhesively bonded together with each adhesive. Specimens were tested in a Hung Ta HT-2402 testing machine at a speed of 2 mm / min. Exemplary results of tensile and compression tests are presented in Fig. 3, and the results of the tests are presented in Table 1.

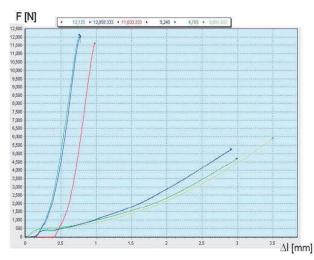


Fig. 3. Comparison of strength of compressed specimens (higher strength) and stretched (lower strength) adhesively bonded with adhesive Epidian57 / Z1

Rys. 3. Porównanie wytrzymałości próbek ściskanych (większa wytrzymałość) i rozciąganych (mniejsza wytrzymałość) klejonych klejem Epidian57/Z1

Table 1. Results of strength tests
Tabela 1. Wyniki badań wytrzymałościowych

Adhesive	Epidian 57/Z1		Poxipol		Dragon		Raychem
Loading	С	S	С	S	С	S	С
Load capacity, N	12125	5245	9623	4028	7777	2718	5713
	12058	4705	9157	4058	8263	2868	6083
	11633	5892	9767	3227	6100	2892	6200
Average load, N	11939	5281	9516	3771	7380	2726	5999

C – compressed specimens, S – stretched specimens

The results of experiment showed a greater than 2.26 to 2.92 strength of compressed specimens compared to stretched ones. This fact allows to conclude that the Mises hypothesis cannot be applied directly to analyze the state of stresses for adhesive joints for all types of adhesively bonded joints or that the strength of the compressed adhesives used for trials is about 2.5 times larger than its tensile strength.

#### **Numerical analysis**

In order to perform numerical calculations, the characteristics of  $\sigma$  =  $\sigma$  ( $\epsilon$ ) of Epidian57 / Z1 adhesive were determined by tensile test of a dog bone specimen with a length of 75 mm measuring base, casted from this adhesive and compression test of a cylindrical specimen with a diameter of 12 mm and a length of 25 mm. Using the extensometer 3542-025M-025-HT2, the value of longitudinal elasticity modulus of the tested adhesive E  $\sim$  2000 MPa was estimated by means of the measurement method. The curves obtained experimentally were converted to real stress curves – cross-sectional deformations of the adhesive were also taken into account (Figure 4). The strength of the compression adhesive was higher than the tensile strength by about 36%.

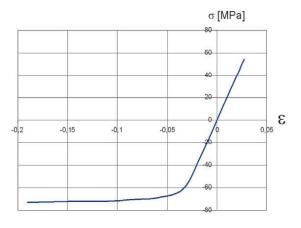


Fig. 4 The curve of actual stress of Epidian 57 / Z1 adhesive hardened at 60oC in 1 hour

Rys. 4 Krzywa naprężeń rzeczywistych kleju Epidian 57/Z1 utwardzanego w temperaturze 60oC w czasie 1 godziny

Numerical calculations were performed in Ansys 16.2 system using the model of the specimen which was loaded with tension and compression with mean values of forces determined experimentally for specimens bonded with Epidian 57 adhesive. The adhesive was molded as an elastic body with a module E = 2000 MPa and a thickness of 0.1 mm. There was a complex stress state in the adhesive layers, but the stress tensor components and redused stresses practically had a constant value on the surface of the adhesive joint (Figures 5, 6, 7).

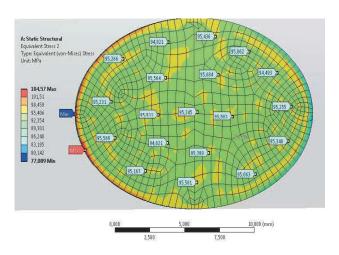


Fig. 5. Distribution of Mises stresses in the adhesive layer of the specimen compressed with 11939 N force

Rys. 5. Rozkład naprężeń Hubera w spoinie klejowej próbki obciążonej na ściskanie siłą 11939 N

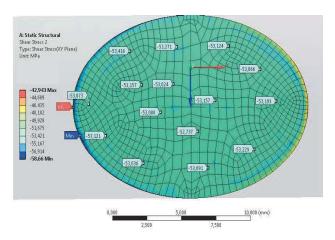


Fig. 6. Distribution of shear stress  $\tau xy$  in the adhesive layer of the specimen compressed with 11939 N force

Rys. 6. Rozkład naprężeń stycznych txy w spoinie klejowej próbki obciążonej na ściskanie siłą 11939 N

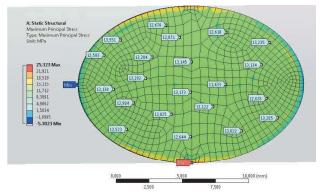


Fig. 7. Distribution of the main maximum stresses in the adhesive layer of the specimen compressed with 11939 N force Rys. 7. Rozkład naprężeń maksymalnych głównych w spoinie klejowej próbki obciążonej na ściskanie siłą 11939 N

Table 2 presents the mean values of stress tensor components and values of reduced and main maximum stresses ( $\sigma$ I) as well as values of minimum stresses ( $\sigma$ III) calculated for destructive loads obtained in the tests: compression and stretching of cylindrical oblique specimens adhesively bonded with Epidian 57.

Table 2. Stress tensor components and reduced stresses calculated for adhesive layer of cylindrical skew joints adhesively bonded with Epidian 57 / Z1

Tabela 2. Składowe tensora naprężeń i naprężenia zredukowane obliczone dla spoin połączeń cylindrycznych skośnych klejonych klejem Epidian 57/Z1

Specimen		Compressed	Tensioned				
Loading, N		11939	5281				
Stresses							
$\sigma_{_{\!\scriptscriptstyle X}}$	MPa	- 29,5	13				
$\sigma_{y}$		- 52,5	23,2				
$\sigma_{z}$		- 27,5	12,4				
T <sub>yz</sub>		53,1	23,3				
T <sub>xy</sub>		0	0				
T <sub>xz</sub>		0	0				
$\sigma_{M}$		95	42,2				
$\sigma_{l}$		12,8	42,1				
σ <sub>III</sub>		-96	-5,6				
T <sub>max</sub>		54,2	24				

The calculations showed, that in the case of compression, von Misses stress is practically equal to the absolute value of the main minimum stresses, and in the case of stretching, the main maximum stresses. The calculated main stresses are correspondingly comparable with the tensile strength of the tested adhesive and to a lesser extent compression. Considering nonlinear adhesive properties described in the curve shown in Fig. 4 in the calculation should reduce the stress value  $\sigma M$ . Such calculations were possible to conduct for a maximum load not exceeding 9000 N. Therefore, the results of calculations in the linear and nonlinear range were compared for this load value (Figures 8 and 9).

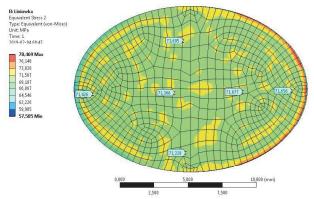


Fig. 8. Mises stress distribution in the adhesive layer of the compression specimen at a load of 9,000 N calculated assuming the nonlinear properties of the adhesive

Rys. 8. Rozkład naprężeń Hubera w spoinie próbki ściskanej przy obciążeniu 9000 N obliczony przy założeniu nieliniowych właściwości kleju

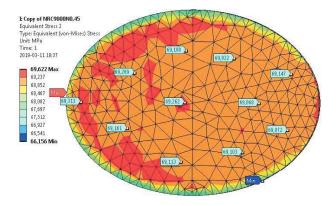


Fig. 9. Mises stress distribution in the adhesive layer of the compression specimen at a load of 9,000 N calculated considering the non-linear properties of the adhesive Rys. 9. Rozkład naprężeń Hubera w spoinie próbki ściskanej przy obciążeniu 9000 N obliczony przy uwzględnieniu nieliniowych właściwości kleju

Taking into account the nonlinear properties of the adhesive resulted in a reduction in the maximum narrative values by about 11% and a more even distribution of stresses in the adhesive layer.

#### Conclusion

- Structural adhesives show higher compressive strength than tensile strength, and thus they exhibit indirect properties between metallic elastic-plastic and brittle ceramic bodies.
- The values of reduced stresses calculated according to the Mises hypothesis in adhesive layers are close to the main maximum stress or the main minimum stresses, but in the latter case they differ by the sign.
- The normal stresses occurring in the joints of the most popular overlapping joints are positive, therefore the Mises reduced stress as well as the main maximum stresses well characterize the effort of such joints.

- The criterion for destroying an adhesive bond loaded temporarily may be that Mises's stress exceeds the absolute value of the tensile strength of the adhesive or compression strength, but it must be considered that these strengths have different values.
- Due to the higher compressive strength of adhesives than tension one, adhesive joints should be designed as much as possible so that their stresses in the adhesive layer are negative stresses.

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